MATLAB Exercise • Level 3

CHARLES J. AMMON
DEPARTMENT OF GEOSCIENCES
PENN STATE UNIVERSITY

P-SV Plane Wave Reflection & Transmission Coefficients

REFLECTION COEFFICIENTS

A common computation in seismology is the calculation of reflection and transmission coefficients that describe the partitioning of energy when a seismic wave strikes a boundary between elastic materials. The reflection coefficient depends on the velocities and densities on either side of the boundary and the ray parameter. The formulas for the values can be found in Table 3.1 of Lay and Wallace (1995) (note that an error in the sign of the second term in the computation of *b*) or in equations 5.38 and 5.39 of Aki and Richards (1980). In the welded elastic boundary, an incident *P* or *SV* wave results in four waves, two reflected and two transmitted or refracted waves. Thus for each case we need four coefficients - notation for each is illustrated in Figure 1.

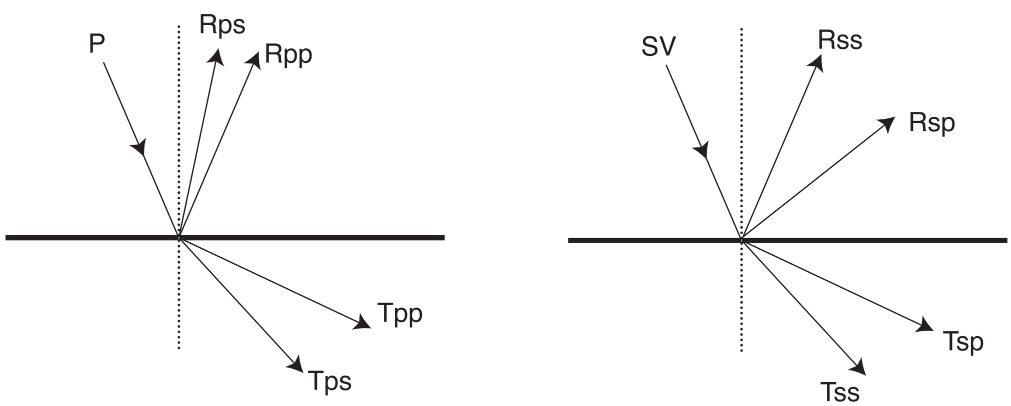

Figure 1. Notation used to identify reflection and transmission coefficients. We must consider two cases, that for an incident P wave and that for an incident SV wave. *R* identifies “Reflected” waves, *T* represents “Transmitted” or “refracted” waves.

The programming of the coefficient equations is straight forward so I won’t go into details. However, the implementation in the script listed below includes one twist. The arguments to the script include the velocity and density values for each material and the ray parameter - which can be a vector. That is, you can call the function with a single value of the ray parameter, or a range of ray parameters stored in a vector. If you pass the function a vector, it will return a complex matrix containing the R/T coefficients *[I thank Dhananjay Kumar, who found a typo in the original version of this script].*

THE P-SV SCRIPT, *PSVRTmatrix*

```
function [RTmatrix] = PSVRTmatrix(p,mi,mt)
%
%    p = ray parameter (a scalar or row vector)
%
%    mi = model of incident    wave [Vp Vs Rho]
%    mt = model of transmitted wave [Vp Vs Rho]
%

% vertical slownesses
etaai = sqrt(1/(mi(1)*mi(1)) - p.*p);
etaat = sqrt(1/(mt(1)*mt(1)) - p.*p);
etabi = sqrt(1/(mi(2)*mi(2)) - p.*p);
etabt = sqrt(1/(mt(2)*mt(2)) - p.*p);
%
a = mt(3)*(1-2*mt(2)*mt(2)*p.*p)-mi(3)*(1-2*mi(2)*mi(2)*p.*p);
b = mt(3)*(1-2*mt(2)*mt(2)*p.*p)+2*mi(3)*mi(2)*mi(2)*p.*p;
c = mi(3)*(1-2*mi(2)*mi(2)*p.*p)+2*mt(3)*mt(2)*mt(2)*p.*p;
d = 2*(mt(3)*mt(2)*mt(2)-mi(3)*mi(2)*mi(2));
%
E = b .* etaai + c .* etaat;
F = b .* etabi + c .* etabt;
G = a - d * etaai .* etabt;
H = a - d * etaat .* etabi;
D = E.*F + G.*H.*p.*p;
%
Rpp = ( (b.*etaai-c.*etaat).*F - (a + d*etaai.*etabt).*H.*p.*p)./D;
Rps = -(2 * etaai .* (a .* b + d * c .* etaat .* etabt) .* p * mi(1)/mi(2) )./D;
Rss = -((b.*etabi-c.*etabt).*E-(a+d.*etaat.*etabi).*G.*p.*p)./D;
Rsp = -(2*etabi.*(a.*b+d*c.*etaat.*etabt).*p*(mi(2)/mi(1)))./D;
Tpp =  (2*mi(3)*etaai.*F*(mi(1)/mt(1)))./D;
Tps =  (2*mi(3)*etaai.*H.*p*(mi(1)/mt(2)))./D;
Tss =   2*mi(3)*etabi.*E*(mi(2)/mt(2))./D;
Tsp =  -2*(mi(3)*etabi.*G.*p*(mi(2)/mt(1)))./D;
%
RTmatrix = [ Rpp' Rps' Rss' Rsp' Tpp' Tps' Tss' Tsp'];
```

EXAMPLE USAGE

Here is an example call of the script that also generates plots similar to those in Lay and Wallace (1995), Figure 3.28 on page 105.

```
% set up the velocity model for two half-spaces
%
%     Vp   Vs   Density
mt = [8.00, 4.6, 3.38 ];  % mt is the half space with transmitted waves
mi = [4.98, 2.9, 2.667];  % mi is the half space with incident wave
%
% generate the ray parameter array
%   the value goes from 0 to 1/Vp(incident)
%
n  = 200
p     = (0:n)*(1/mi(1))/n;
%
R = PSVRTmatrix(p,mi,mt);
%
```

```
Rpp = R(:,1); % P-to-P reflection
Rps = R(:,2); % P-to-S reflection
Tpp = R(:,5); % P-to-P transmission
Tps = R(:,6); % P-to-S transmission
%
%  The rest is graphics
%
% compute incidence angle in degrees
%
angle = (180/pi) * (asin(p*mi(1)));
%
ymin = -.1; ymax = 2.0;
xmin = 0; xmax = 90;
%
subplot('position',[0.1 0.6, 0.35 0.3])';
plot(angle,abs(Rpp),'k-')
xlabel('Incidence Angle (°)');
ylim([ymin ymax]); xlim([xmin xmax]); grid on;
title('Reflected P');
%
subplot('position',[0.55 0.6, 0.35 0.3])';
plot(angle,abs(Rps),'k-')
xlabel('Incidence Angle (°)');
ylim([ymin ymax]); xlim([xmin xmax]); grid on;
title('Reflected S');
%
subplot('position',[0.1 0.1, 0.35 0.3])';
plot(angle,abs(Tpp),'k-')
xlabel('Incidence Angle (°)');
ylim([ymin ymax]); xlim([xmin xmax]); grid on;
title('Transmitted P');
%
subplot('position',[0.55 0.1, 0.35 0.3])';
plot(angle,abs(Tps),'k-')
xlabel('Incidence Angle (°)');
ylim([ymin ymax]); xlim([xmin xmax]); grid on;
title('Transmitted S');
```

The results of running the script are shown in Figure 2. Note the large changes in the coefficients when the P-wave becomes critical.

EXERCISES

Exercise 1: Compute and plot of the reflection coefficients for a P-wave incident from a elastic solid with a P-wave velocity of 3.0 *km/s*, an S-velocity of $\sqrt{3}$ *km/s*, and a density of 2.67 g/cm^3 striking a material with a P-wave velocity of 5.8 *km/s*, an S-velocity of $5.8/\sqrt{3}$ *km/s*, and a density of 2.67 g/cm^3.

Exercise 2: Create a plot of the amplitude and the phase of a P-wave reflected at a boundary between two Poisson solids with P-velocities of 6.5 and 8.0 km/s and densities of 2.80 and 3.33 g/cm^3.

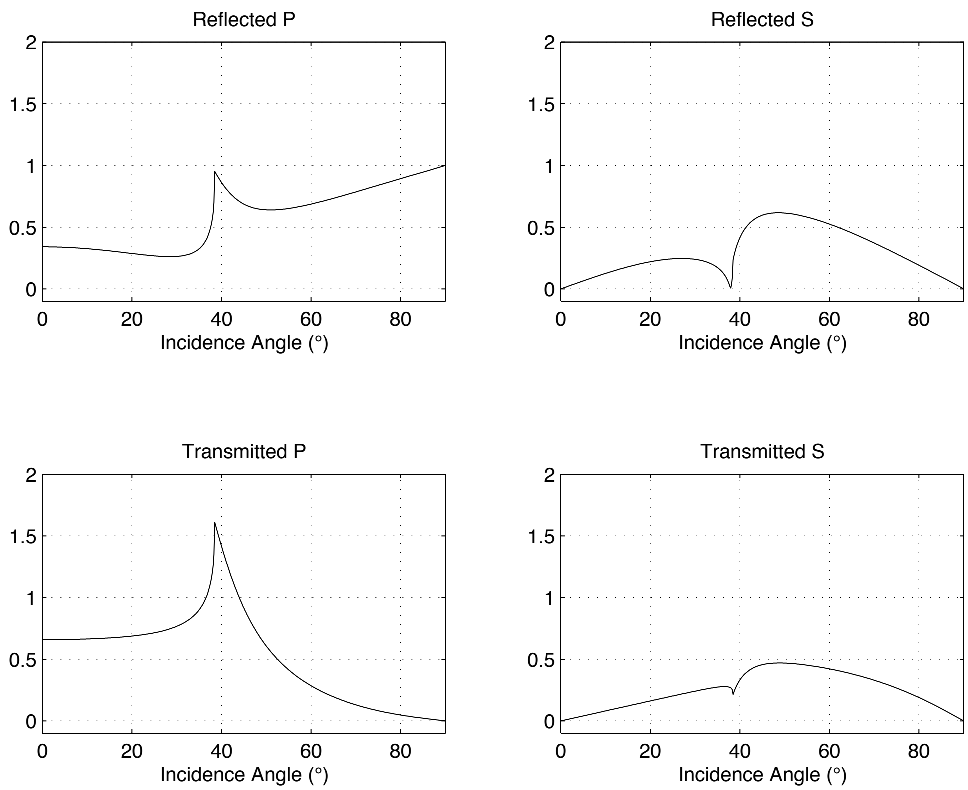

Figure 2. Results of running the script with the parameters used in Lay and Wallace (1995) Figure 3.28. The critical angle of the P-wave is 38.5°. Beyond this angle the transmission coefficient is complex.

Exercise 3: Create a plot of the reflection and transmission coefficients for the model used in Figure 2 but for an incident SV-wave. Identify and explain the regions where the coefficients experience a rapid variation.

Exercise 4: Modify the script to handle the SH-wave reflection and transmission coefficients. Produce a plot for an SH wave incident on elastic boundaries with transitions from low to high velocity and high to low velocity.

Exercise 5: Modify the script to compute the free-surface *P-SV* reflection coefficients for a range of ray parameter values. Make a plot of the reflected *P* and *SV* wave amplitudes for a range of incidence angles of a *P* wave and then an *SV* wave.

Exercise 6: Plot the amplitude and the phase of the P- and SV-wave reflections for the example shown in Figure 2. The results are shown in Figure 3. Give a physical interpretation of the phase shifts - particular values equal to 180°.

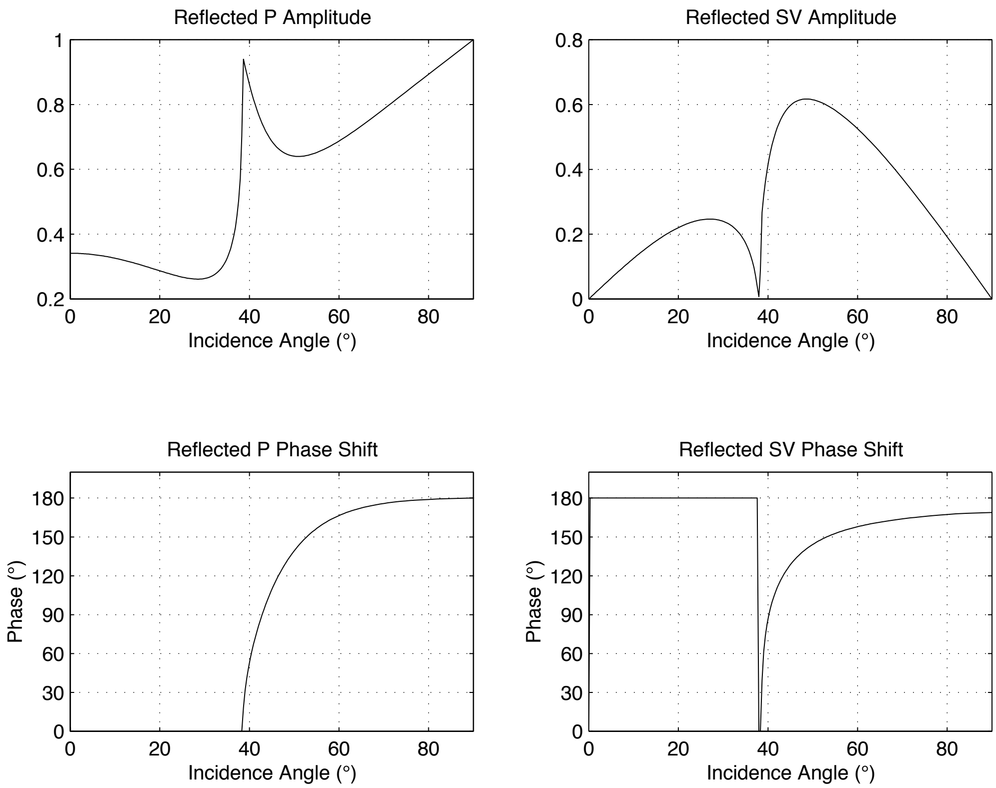

Figure 3. Amplitude and phase of reflected P and SV waves for the parameters used in Figure 2. Note the change in phase at the critical angle, 38.5°. The phase of the SV wave for vertical incidence is inconsequential since the *Rps* reflection coefficient is zero. The phase shifts are changes relative to the phase of the incident P wave.