

MATLAB Exercise • Level 1

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Computing the Pressure within Earth

COMPUTING PRESSURE WITHIN EARTH

One of the fundamental parameters in any geologic environment is the pressure. Pressure plays a principal role in the mineralogy of rocks which affects the other observable qualities such as porosity, density, or seismic velocity. Computing the pressure within the planet is relatively easy if you know the density. We'll adopt a local Cartesian reference frame where the structure depends only on depth. This is not a bad approximation for most pressure calculations.

Computation of the pressure within a column of known density, $\rho(z)$, requires the evaluation of an integral

$$P(z) = \int_0^z (\rho(z) \cdot g) dz \quad (1)$$

where g is the acceleration of gravity. Near Earth's surface we can assume that g is roughly constant (9.80 cm/s^2), but if we did a global problem where we looked at the pressure deep inside Earth, we would have to also consider $g \rightarrow g(z)$. The units of density are g/cm^3 or kg/m^3 ; to convert from CGS to MKS, multiply the numerical value of the density by 1000 (e.g. $2.67 \text{ g/cm}^3 = 2,670 \text{ kg/m}^3$).

Within a region of uniform density, the pressure is given by

$$\begin{aligned} P(z) &= \int_{z_{top}}^z (\rho_0 \cdot g) dz \\ &= \rho_0 \cdot g \cdot \int_{z_{top}}^z dz \\ &= g \cdot \rho_0 \cdot z \Big|_{z_{top}}^z \\ &= g \cdot \rho_0 \cdot (z - z_{top}) \end{aligned} \quad (2)$$

That's a simple computation. The only caution is that you have to be careful that the units of all the parameters (g , ρ , z) are consistent.

SCRIPT *epressure_constant_d(zmax,density)*

The script listed below, *epressure_constant_d*, is a direct computation of the approximate pressure in the crust using equation (2) and a uniform density. We'll make the density a parameter of the script so that we can change it and see the difference in pressure for different densities. The results are returned in mega-Pascals (MPa). If you need the values in another set of units, you can use the following conversion factors:

1 kilobar	100 MPa
1 atmosphere	101,325 Pa

Here's the script:

```
function [ P, Z ] = epressure_constant_d(zmax, density)
%
% Function to compute pressure increase within a region of
%   constant density. Z increases downward.
%
% INPUT:
%   zmax   = bottom of region   (in km)
%   density = density in g/cm^3
%
% OUTPUT:
%   P = the pressure at the top and bottom of the region
%   Z = the depths of the top & bottom of the region
%
% convert velocity & density to MKS units
% this does not affect vs in the calling function
density = density * 1000;
zmin    = zmin * 1000;
zmax    = zmax * 1000;
%
g = 9.8e-02; % m/s^2
%
P(1) = 0;
Z(1) = 0;
P(2) = zmax * g * density;
Z(2) = zmax;
```

The script returns the pressure in Pascals and depth in kilometers. Here's a comparison of the pressure in the outer 10 km on Earth with and without the crust

```
>> clf
>> [PV ZV] = epressure_constant_d(10,3.20);
>> [PE ZE] = epressure_constant_d(10,2.67);
>> h = axes('position', [.1, .1, 0.5, .8]);
>> plot(PE/1e6,ZE/1e3,'k-', PV/1e6,ZV/1e3,'k-')
>> set(h,'FontName','Helvetica','FontSize',14);
>> set(h,'Ydir','reverse')
```

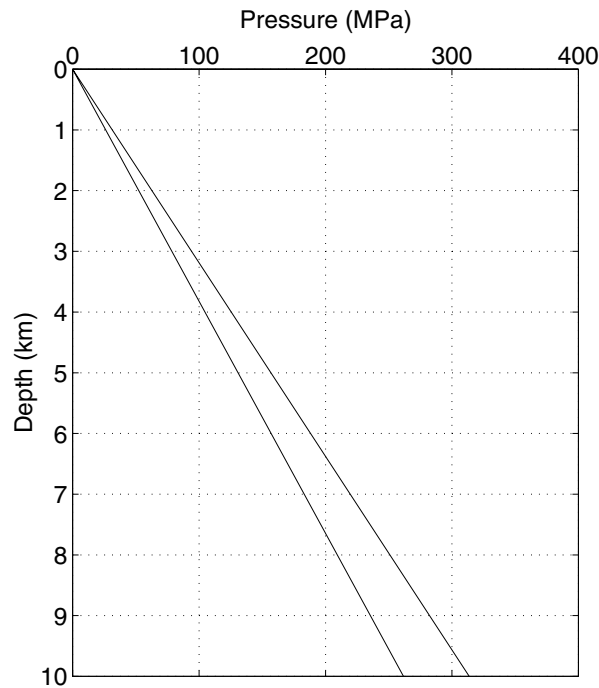


Figure 1. Pressure versus depth for two densities, one corresponding to typical continental crust, the other to mantle.

```
>> set(h, 'FontName', 'Helvetica', 'FontSize', 14);
>> set(h, 'XaxisLocation', 'top')
>> xlabel('Pressure (MPa)')
>> ylabel('Depth (km)')
>> grid on
>>
```

Most of the commands in the application were set to adjust the graphics. MATLAB can do very nice graphics, but you have to execute commands to do it. Some times that is inconvenient, but often it lets you easily produce many plots with a uniform “look”.

EXERCISES

Exercise 1: The scripts don't include the pressure of Earth's atmosphere, which is about 1×10^5 Pa. Does this make a significant difference in most geological problems? How deep into Earth is it before you double the atmospheric pressure?

Exercise 2: What is the pressure gradient in the shallow part of Earth if the density is 2.67 g/cm^3 .

Exercise 3: Write a MATLAB script to compute the pressure in the shallow regions of Venus, where the gravitational acceleration is approximately 8.60 m/s^2 but the atmospheric pressure is 9×10^6 Pa.

Exercise 4: Assume that because of compaction, the density in sediments increases linearly with depth. Use (1) to derive an analytical expression for the density and then modify the first script to compute the pressure within a column of self-compacting sediments.

Exercise 5: Write a script to compute the pressure for a model that consists of a set layer densities and thicknesses