MATLAB Exercise • Level 1

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Great-Circle Arc Distances

COMPUTING GREAT-CIRCLE DISTANCES ON A SPHERE

For many computations in seismology we need to have the angular distance between two points on the surface of Earth. Applications include estimating earthquake wave arrival times that are usually shown on travel time curves that give the distance from the earthquake to the seismometer in distance measured in angular degrees. For work concerning Earth, we usually use an idealized sphere that has the same volume as the planet. The radius of our idealized planet is approximately 6,370.8 km (you should memorize ~ 6371 km).

Converting from Latitude and Longitude to a Position Vector

To compute the angular distance between two points on the sphere we'll use vector algebra. Generally we refer to points on Earth's surface by the latitude, λ and longitude, ϕ. From these values we can construct Cartesian "position" vectors using:

$$\begin{aligned} A_x &= \cos\lambda \cdot \cos\phi \\ A_y &= \cos\lambda \cdot \sin\phi \\ A_z &= \sin\lambda \end{aligned} \tag{1}$$

The x-component lies in the plane of Earth's equator and points toward a longitude of 0°, the y-component is in the equatorial plane and points towards 90° East, and the z-component points towards the North Pole. The position vector points from the center of the sphere towards the point on Earth's surface.

Computing the Distance Using the Scalar Product

The get the angular distance between two points on the surface, convert each to a vector and then compute the scalar product of the two vectors.

$$\vec{A} \cdot \vec{B} = |A| \cdot |B| \cdot \cos\theta = A_x B_x + A_y B_y + A_z B_z . \tag{2}$$

And so the angular distance (in radians) is given by

$$\Delta = \operatorname{acos}(\vec{A} \cdot \vec{B}) . \tag{3}$$

Implementing the Calculation in MATLAB

The implements follows our discussion - convert the latitude and longitude to vectors, compute the vector cross product and then take the inverse cosine of the result. To convert to degrees, simply multiply the answer by $180/\pi$. Here's a list of MATLAB commands to evaluate the distance from (45N,-90E) to (70N,90N). The answer is obviously 25°, since the two points lie along the same meridian.

```
>> deg_to_rad = pi / 180;
>> lat1 = 45*deg_to_rad;
>> lon1 = -90*deg_to_rad;
>> lat2 = 70*deg_to_rad;
>> lon2 = -90*deg_to_rad;
>> P1 = [cos(lat1)*cos(lon1), cos(lat1)*sin(lon1),sin(lat1)];
>> P2 = [cos(lat2)*cos(lon2), cos(lat2)*sin(lon2),sin(lat2)];
>> Gcarc = acos(sum(P1 .* P2)) * 180 / pi
>> Gcarc
>> Gcarc =
>>   25.00000000000001
```

Making a Reusable Script to Compute Distances

As you might imagine, distance calculator is a frequently needed tool, so it's worth some time to convert the above theory into a simple little function. Here's the version I designed.

```
function [ gcarc ] = gcarc(lat1,lon1,lat2,lon2)
%
%   Compute the angular distance between two points on a sphere.
%
%   INPUT:
%
%         lat1, lon1 latitude and longitude of point 1 (in degrees)
%         lat2, lon2 latitude and longitude of point 2 (in degrees)
%
%   OUTPUT:
%
%         gcarc = the angular distance in degree
%

% convert from degrees to radians (original values are unchanged)
deg_to_rad = pi / 180;
lat1 = lat1*deg_to_rad;
lon1 = lon1*deg_to_rad;
lat2 = lat2*deg_to_rad;
lon2 = lon2*deg_to_rad;
%
% convert to Cartesian Coordinates
%
P1 = [cos(lat1)*cos(lon1), cos(lat1)*sin(lon1),sin(lat1)];
P2 = [cos(lat2)*cos(lon2), cos(lat2)*sin(lon2),sin(lat2)];
%
% compute the angle between the vectors using the scalar product
%
gcarc = acos(sum(P1 .* P2)) * 180 / pi;
```

To use the script, place it in a directory contained in your MATLAB path, and then you can treat it as any other MATLAB command

```
>> gcarc(45, -90, 65, -90)
>> ans =
>>   20.00000000000001
```

In MATLAB, the initial comments (all those after the function line and before the first blank line) serve as a “help page”, so make them thorough enough for you to remember how to use the function.

```
>> help gcarc
>>
>>
>>     Compute the angular distance between two points on a sphere.
>>
>>     INPUT:
>>
>>           lat1, lon1 latitude and longitude of point 1 (in degrees)
>>           lat2, lon2 latitude and longitude of point 2 (in degrees)
>>
>>     OUTPUT:
>>
>>           gcarc = the angular distance in degree
```

EXERCISES

Exercise 1: Implement the above script and use it to estimate the distance between the following locations (North and East are positive):

45N,18E	23S, 45W
90N, 32E	0N, 15E

Exercise 2: Considering the fact that the radius of Earth is about 6371 km, what distance in kilometers does one degree represent? (Hint: the circumference of Earth is $2\pi \cdot radius$).

Exercise 3: A long-period Rayleigh wave travels across Earth’s surface with a group velocity of 3.65 km/s. How long after an earthquake in California does it take the Rayleigh wave to reach St. Louis, MO? Tokyo, Japan? All the way around the world and back to California?

Exercise 4: Look through books on geodesy to find the equations for computing distance on an ellipsoidal surface with the shape of Earth and write a script to compute more accurate distances.