Notes on Seismic Surface-Wave Processing
Part I
• Group Velocity Estimation •

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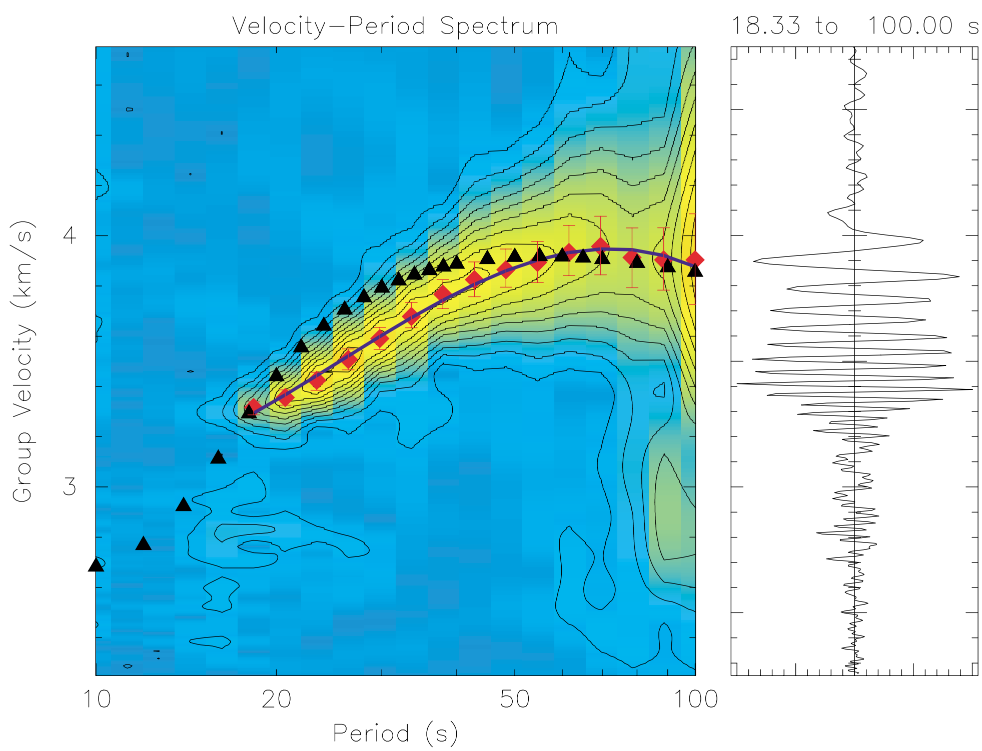

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1 • Introduction

Many steps are involved in extracting dispersion information from raw seismograms. The basic theory in the processing is documented in some of the classic papers of seismology. This is neither a text book nor a tutorial, these are research notes that I created for myself and my students. The final references for the techniques are available in the published seismological literature. I compiled these notes to collect the details used to process surface waves for tomographic studies into a single document that I can use for reference. Included are listings of c-shells, *awk* commands, SAC macros, some theory and some algorithms, as well as reference information on moment-tensor conventions and surface-wave dispersion. These are notes, not a formal publication, and I have not meticulously cited the literature throughout the text but I have drawn on published works on signal processing and surface-wave analysis. Works that were particularly valuable to me included (in alphabetical order):

> Claerbout, J., F., Earth Soundings Analysis: Processing Versus Inversion, Blackwell Scientific Publications, Boston, MA, 304 pages, 1992.
>
> Dziewonski, A., S. Bloch and L. Landisman, A technique for the analysis of transient seismic signals, Bull. Seism. Soc. Am., 59, 427-444, 1969.
>
> Keilis-Borok, V. I., Seismic surface waves in a laterally inhomogeneous earth, Kluwer Academic Publishers, Dordrecht, 293 pages, 1989.
>
> Levshin, A., L. Ratnikova and J. Berger, Peculiarities of surface-wave propagation across central Asia, Bull. Seism. Soc. Am., 82, 2464-2493, 1992.

For a nice review of what can be done with group velocity measurements, I strongly recommend:

> Ritzwoller, M. H., and A. L. Levshin, Eurasian surface wave tomography: Group velocities, J. Geophys. Res., 103, 4839-4878, 1998.

In addition to the above there are many exemplary surface-wave analyses described in the seismological literature. I encourage the reader to invest substantial time surveying the earlier works (start with Ritzwoller and Levin (1998) and follow the reference threads - you'll find much to read).

Surface-Wave Dispersion

The curvature of the Earth and the variation of seismic velocities with depth in the Earth cause different components of a surface wave to travel with different velocities and so as they propagate, surface waves disperse. Generally, the shorter period surface waves, which sample the rocks closest to the surface, travel slower than the longer period waves which are sensitive to the faster wave velocities found deeper in the Earth. Thus, the farther a surface wave travels, the longer its vibrations last (the amplitude decreases primarily from geometric spreading and attenuation, but dispersion affects the peak amplitude of the signal as well). Dispersion is quantified by specifying one or two velocities (or one velocity and a constant) for each frequency of interest. The two velocities are called phase and group velocities. Both can be measured from seismograms and both are sensitive to the structure of the rocks through which the surface waves travel. The group velocity is the velocity at which the energy travels and can be used to predict the arrival time of surface-wave vibrations of a particular frequency. The phase velocity describes the phase of the vibration at a particular distance from the source of excitation and is sensitive to the source depth and faulting geometry. The group velocity is often represented using the symbol $U(\omega)$, the phase velocity using the symbol $c(\omega)$, where ω is the angular frequency. Lay and Wallace (1995) show that the group and phase velocities are related by:

$$U(\omega) = \frac{d\omega}{dk} = \frac{d\{kc(\omega)\}}{dk} = c(\omega) + k \cdot \frac{dc(\omega)}{dk} \qquad (1\text{–}1)$$

where we have used the fact that $c(\omega) = \omega/k$ and k is the wavenumber. An example of the period-dependent velocities is shown in Figure 1–1.

The information in the two dispersion measurements is not independent, although in the presence of noise, measuring both is a good idea. We can use check the group and phase velocities estimated from a seismogram for consistency. The check can be important, since the phase velocity estimated using a single-station measurement requires that we know an initial source phase, which depends on the source type (earthquake, explosions, etc.) and depth, and on the source receiver geometry (azimuth relative to strike for an earthquake source). A useful equation for checking the consistency of phase and group velocity estimates is

$$U(\omega) = \left[\frac{1}{c} - \frac{\omega}{c^2} \cdot \frac{dc}{d\omega}\right]^{-1} \qquad (1\text{–}2)$$

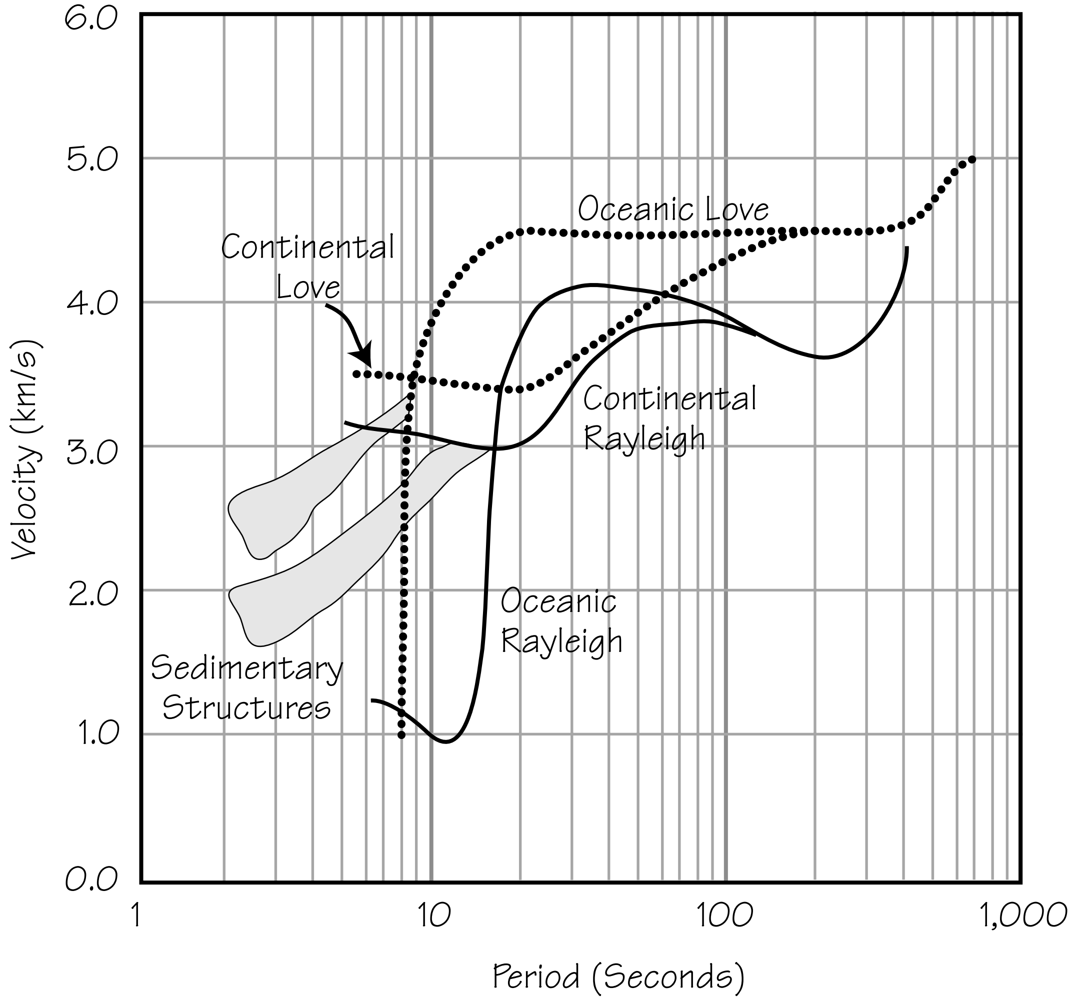


Figure 1–1. Sample dispersion curves based on observations. The solid lines show the group velocity, the dotted lines show the phase velocity. Note the large differences at short periods between waves traveling along paths through the oceans and those traveling across the continents. Shallow geology can have a big impact on dispersion at short periods, and the effects of sedimentary basins is shown as shaded regions at short periods.

Some Surface-Wave Terminology

An important part of measuring surface-wave dispersion is the estimation of what's real in a signal and what is artifact of ground noise, glitches, *etc*. As they propagate, surface waves average many of the details in the earth structure and their dispersion curves are generally smooth. Values at long periods should approch the values of any decent reference model since variations in Earth structure measured at these long periods usaully range over a few percent.

In general, the more you understand about surface-wave theory and the more experience that you have, the easier it is to make the dispersion measurements. When discussing you measurements, it will be esier if you master the terms and concepts that are common in the description and discussion of surface waves. I've listed a few in Table 1-3.

Table 1-3: Surface-Wave Terminology and "Rules of Thumb"

Term or Observation	Comment
Normal Dispersion	Velocity increases with period
Reverse Dispersion	Velocity decreases with period.
Airy Phase	A region of the curve where the derivative of the group velocity vanishes. The amplitudes of Airy phases are relatively large.
Multipathing	When the structure is heterogeneous more than one "packet" of surface waves may be recorded at a particular location. The problem can be severe when the "direct" wave packet is nearly nodal because of radiations patterns or source depth.
Beating	When the signal "beats" two arrivals with similar velocities are interfering, possibly from off-azimuth propagation such as multipathing.
Spectral hole or Spectral notch	A region of the surface wave spectrum where the amplitude decreases rapidly. Notches can be caused by source depth (Rayleigh waves) or by interference multipathed arrivals.
Rayleigh-wave "leakage"	When Love waves are "nodal", watch for Rayleigh waves "leaking" onto the transverse component. The problem of Love wave leakage is less trouble, because the Love waves don't leak onto the vertical (anisotropy excepted).
Ground noise	Noise is generally larger on the horizontal seismometers than the vertical and larger at short-periods (near microseisms and the longest periods). Small events seldom produce long-period signals above the noise level.
Observed variations in dispersion.	Since earth structure is relatively simple for long-period waves (>100-150 s) the variations in structure should not be as large in the long-period band than the short-period band.

2 • Seismogram Preparation

Examining Your Signals

The first step in processing the seismograms for each earthquake is examine the quality of the signal. One of the problems with surface-wave studies is background seismic “noise” such as microseisms or body waves that arrive at the same time as the surface waves. The best way to assess the affect of noise in a signal is to look at the seismograms over a range of frequencies and so plotting the observations is a good first step in surface-wave analysis. I do all my data inspection using SAC and SAC macros. The first step is to mark a wide group velocity range and lot the observed signals in several bandwidths.

The SAC macro called “*pltevent.macro*”, used to filter and plot the observed seismograms in four frequency ranges, is listed below and two examples of the results of running *pltevent.macro* are shown in Figures 2–2 and 2–3. The origin time and location (latitude and longitude) of the earthquake must be entered into the SAC header before proceeding. This information is contained in the file yymmdd.loc, that is obtained with the data. Here is a listing of the SAC macro *pltevent.macro:*

```
fileid l ul
fileid t l kcmpnm
fileid on
title on
gt s si t
qdp 1200
rh $1$.LHZ
*
* MAKE A NICE TITLE
*
evaluate to c &1,dist + 0.5
evaluate to r ( INT %c )
setbb rs ( before '.' ' %r ' )
evaluate to c &1,az + 0.5
evaluate to a1 ( INT %c )
setbb a2 ( before '.' ' %a1 ' )
title '( concatenate' &1,kstnm ' ', r = '  ' %rs km ' ', az = ' ' %a2 ' ) '
xlabel "Time @(s@)"
ylabel "Velocity @(counts@)"
*
ylim off
*
* Set up a group velocity Window
*
evaluate to tmax  &1,dist / 2.0 + &1,o
evaluate to tmin  &1,dist / 25.0 + &1,o
```

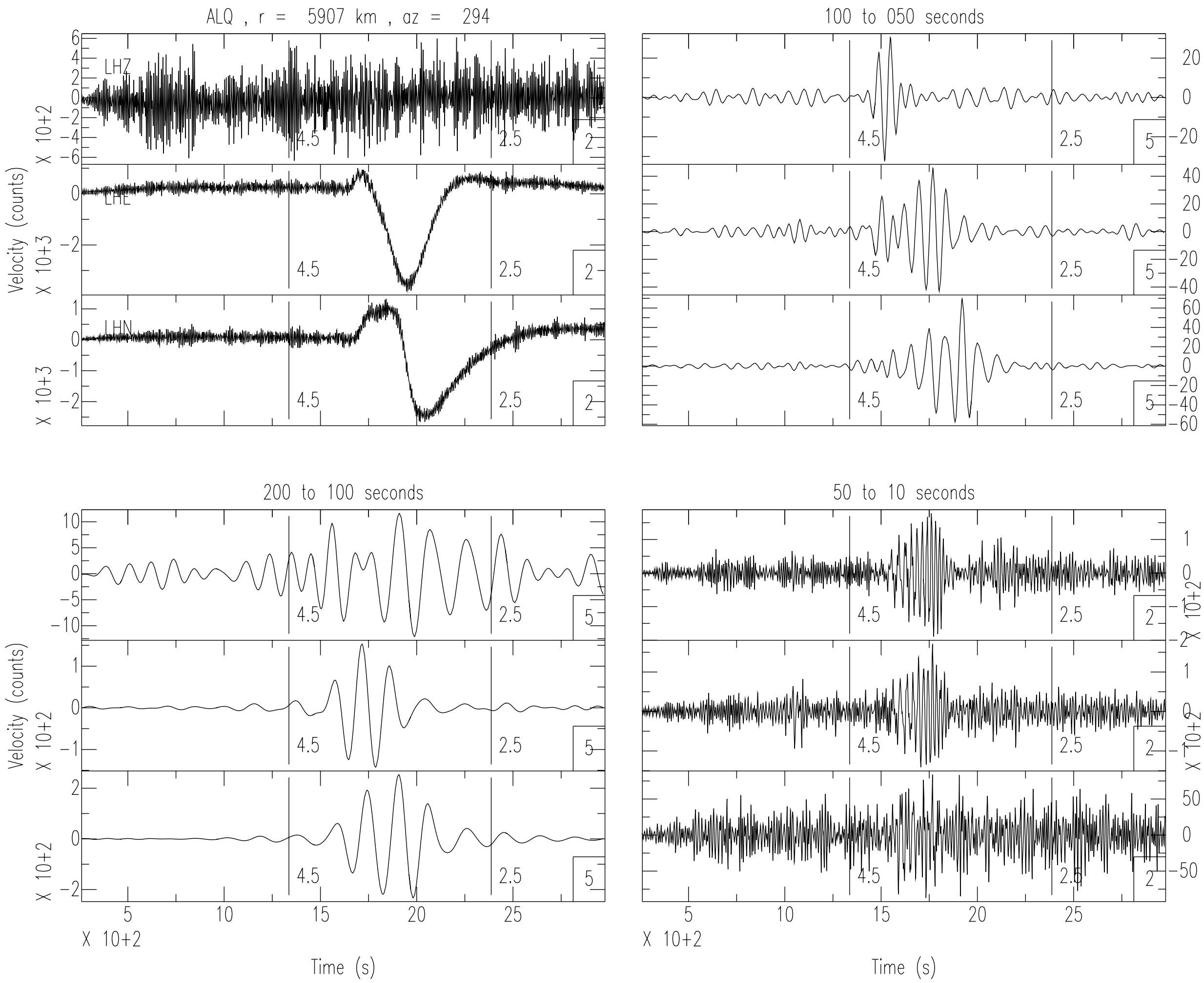


Figure 2–2. Sample raw data from station ALQ, Albuquerque, NM. The panel in the upper left is unfiltered and the title in that panel lists the distance (r) and the source-to-station azimuth. The labeled vertical lines show indicate group arrival times for the velocities shown next to the line. The other three panels show the results of bandpass filtering the seismograms using a four-pole, two-pass Butterworth filter into different period ranges. The corner periods of the filters are shown above each panel. The data units are counts, which for most sensors indicates that a velocity seismogram is shown. This example was chosen to illustrate potential problems with blindly processing signals without any visual inspection. The horizontal components are contaminated with a large transient noise pulse. Narrow-band filtering masks the long-period problems with the original seismograms and makes it harder to identify the problem.

```
*
xlim %tmin %tmax
*
* BEGIN THE PLOT
*
begfr
axes on l
xlabel off
*
* FRAME 1 - full bandwidth
*
xvp 0.15 0.5
```

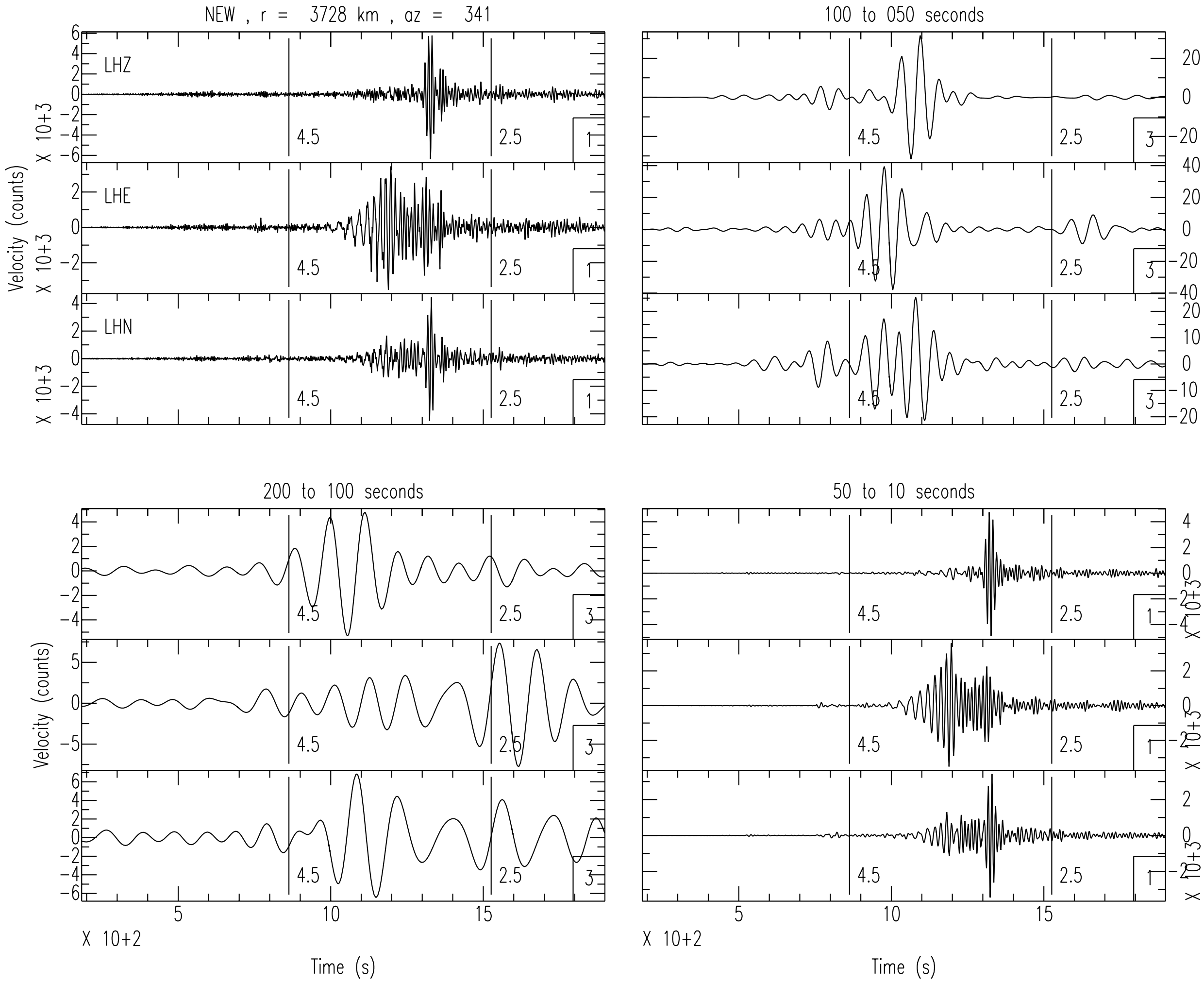


Figure 2–3. Example processing of an event from the Middle America Trench (04/23/96 06:53:35, mb = 5.3) recorded at station NEW, Newport Washington. The panel in the upper left is unfiltered and the title in that panel lists the distance (r) and the azimuth of the seismometer. The other three panels show the results of band-pass filtering the seismograms using a four-pole, two-pass Butterworth filter into different period ranges. The corner periods of the filters are shown above each panel. The data units are counts, which for most sensors indicates that a velocity seismogram is shown.

```
yvp 0.575 0.925
r $1$.LHZ $1$.LHE $1$.LHN
markt v 4.5 2.5
rmean
taper w .2
p1
*
* FRAME 2 - 200 to 100 seconds
*
fileid off
axes on l b
xlabel on
yvp 0.15 0.5
bp bu c 0.005 0.01 n 4 p 2
title "200 to 100 seconds"
```

```
p1
*
* FRAME 3 - 100 to 50 seconds
*
axes on r
ylabel off
xlabel off
xvp 0.525 0.875
yvp 0.575 0.925
title "100 to 050 seconds"
r $1$.LHZ $1$.LHE $1$.LHN
markt v 4.5 2.5
rmean
taper w .2
bp bu c 0.01 0.02 n 4 p 2
p1
*
* FRAME 4 - 50 to 10 seconds
*
axes on r b
xlabel on
yvp 0.15 0.5
r $1$.LHZ $1$.LHE $1$.LHN
rmean
taper w .2
bp bu c 0.02 0.1 n 4 p 2
title "50 to 10 seconds"
markt v 4.5 2.5
p1
*
* END of PLOT
*
endfr
ylim off
title off
```

Windowing the Surface-Waves

The next step in the procedure is to remove the effects of the instrument response from the seismograms. Again, I use SAC for this purpose. To limit storage requirements, only that portion of the initially long time window is cropped and used in further processing. The window selected using a group-velocity range and the event location and origin time are required to compute the start and end times of the window. The formulas are quite simple:

$$t_{start} = t_o + \frac{\Delta}{v_{max}} \tag{2–4}$$

and

$$t_{end} = t_o + \frac{\Delta}{v_{min}} \tag{2–5}$$

where Δ is the source-receiver distance and t_0 is the event origin time. The SAC command to do this automatically is call MARKT, and the minimum and maximum group velocities must be specified in the macro listed below, but since the windowing is combined with instrument response deconvolution, I describe that before listing the macro.

Removing the Instrument Effects

To recover the ground displacement, we must deconvolve the instrument response from the signals. To compute the instrument response, a file that contains the poles and zeros describing the response must be created. The pole-zero files are sent with each set of observations so the information for computing the instrument response is located in the directory "*Resp*" located above the raw data, which are in directories "Usnsn" or "CNSN". The instrument response is computed using

$$I(\omega) = \frac{\prod_{k=1}^{nz} (i\omega - z_k)}{\prod_{k=1}^{np} (i\omega - p_k)} \tag{2–6}$$

where *np* and *nz* are the number of poles and zeros respectively; and p_j and z_i are the poles and zeros. Note that the units of the poles and zeros are radians. In addition to the poles and zeros, you must also specify four frequency values that define a cosine window applied to the instrument deconvolution result. The window does not distort the phase and is designed to remove numerical noise caused by dividing the seismogram spectrum by very small values of the instrument response. Generally, instrument responses are very small at the low and high frequency limits. You should adjust the filter parameters depending on the size of the earthquake that generated the signal. Larger events will produce better long-period signals. In the example below, the windows are set for a small earthquake that is unlikely to generate large signals for periods greater than approximately 150 seconds. So, the Fourier Transform of the displacement is represented by

$$D(\omega) = T(\omega) \cdot \frac{S(\omega) I^*(\omega)}{I(\omega) I^*(\omega)} \tag{2–7}$$

where $T(\omega)$ represents the cosine taper and * indicates complex conjugation. Be careful how you use the frequency-domain taper, which is an acausal filter. When you remove instruments with SAC, this filter remains in your displacement seismograms (and if you remove instruments effects another way, something equivalent to the taper will be present). If you are estimating group or phase velocities the zero phase of the filter will not affect the results. If you are using the seismograms or the amplitudes, you may run into complications if you analyze the harmonics that lie within the frequency bands that are attenuated by the taper. In theory, you should apply the same filter to any signals that you compare with these displacements. At a minimum, keep track of what frequencies you used to define you frequency domain taper. Here is the macro that I use:

```
*******************************************************************
*
*  REQUIRES TWO ARGUMENTS: STATION_NAME 'all'
*  if $2 == 'all' it will look for horizontals (otherwise, it won't)
*
*  THE ORIGINAL DATA   FILES ARE STORED IN ../Usnsn[Cnsn]
*  THE POLES AND ZEROS FILES ARE STORED IN ../Resp
*
*  OUTPUT IS DISPLACEMENT IN CM
*
*  NOTE THE FREQUENCY-DOMINA TAPER CORNERS!
*
*******************************************************************
r ../Usnsn/$1$.LHZ
*
* SET UP THE FREQUENCY WINDOW
*    200 seconds to 166.7 seconds
*
setbb f1 0.005
setbb f2 0.006
setbb f3 45
setbb f4 55
*
rmean
taper
*
* DO THE VERTICAL
*
transfer from polezero subtype ../Resp/$1$.LHZ.RESP freq %f1 %f2 %f3 %f4
w $1$_dsp.z
*
* FOR HORIZONTALS
* KEY ON COMMAND LINE ARGUMENT
*
if $2 eq "all"
*
r ../Usnsn/$1$.LHN
rmean
taper
transfer from polezero subtype ../Resp/$1$.LHN.RESP freq %f1 %f2 %f3 %f4
w $1$_dsp.n
*
r ../Usnsn/$1$.LHE
rmean
taper
transfer from polezero subtype ../Resp/$1$.LHE.RESP freq %f1 %f2 %f3 %f4
w $1$_dsp.e
*
r $1$_dsp.n
ch cmpaz 0
ch cmpinc 90
wh
*
```

```
r $1$_dsp.e
ch cmpaz 90
ch cmpinc 90
wh
*
r $1$_dsp.n $1$_dsp.e
rotate
ch kcmpnm Radial
w $1$_dsp.r $1$_dsp.t
r $1$_dsp.t
ch kcmpnm Trnsvrs
wh
*
endif
*
* END of HORIZONTALS PROCESSING
*
*
*   WINDOW USING GROUP VELOCITIES
*
*   VMAX = inf
*   VMIN = 2.0 km/s
*
evaluate to t0 &1,o
evaluate to t1 &1,dist / 2.0 + %t0
*
cut %t0 %t1
r $1$_dsp.z $1$_dsp.r $1$_dsp.t
*
* CONVERT to CM
*
mul 100
*
w over
*
cut off
*
sc /bin/rm $1$_dsp.n $1$_dsp.e
```

The amplitudes of the signals are first converted to meters, and then scaled by a factor of 100, producing displacement in *cm*. Note that the last segment of the script cuts out that part of the signal that would arrive at the station with a group vleocity greater than 2.0 *km/s*. The start of the resulting trace is the event origin time. An example of the results of the macros “correct.mac” and pltdsp.macro is shown in Figure 2–4.

A Script to Build the Macros

To construct macros to “correct” a set of observations, the following c-shell is useful.

```
mkdir Disp
cd Disp
#
echo 'Making scripts'
#
pwd
sed  -e s%../../pltevent%correct% ../Usnsn/plt.all.macro >! tmp
```

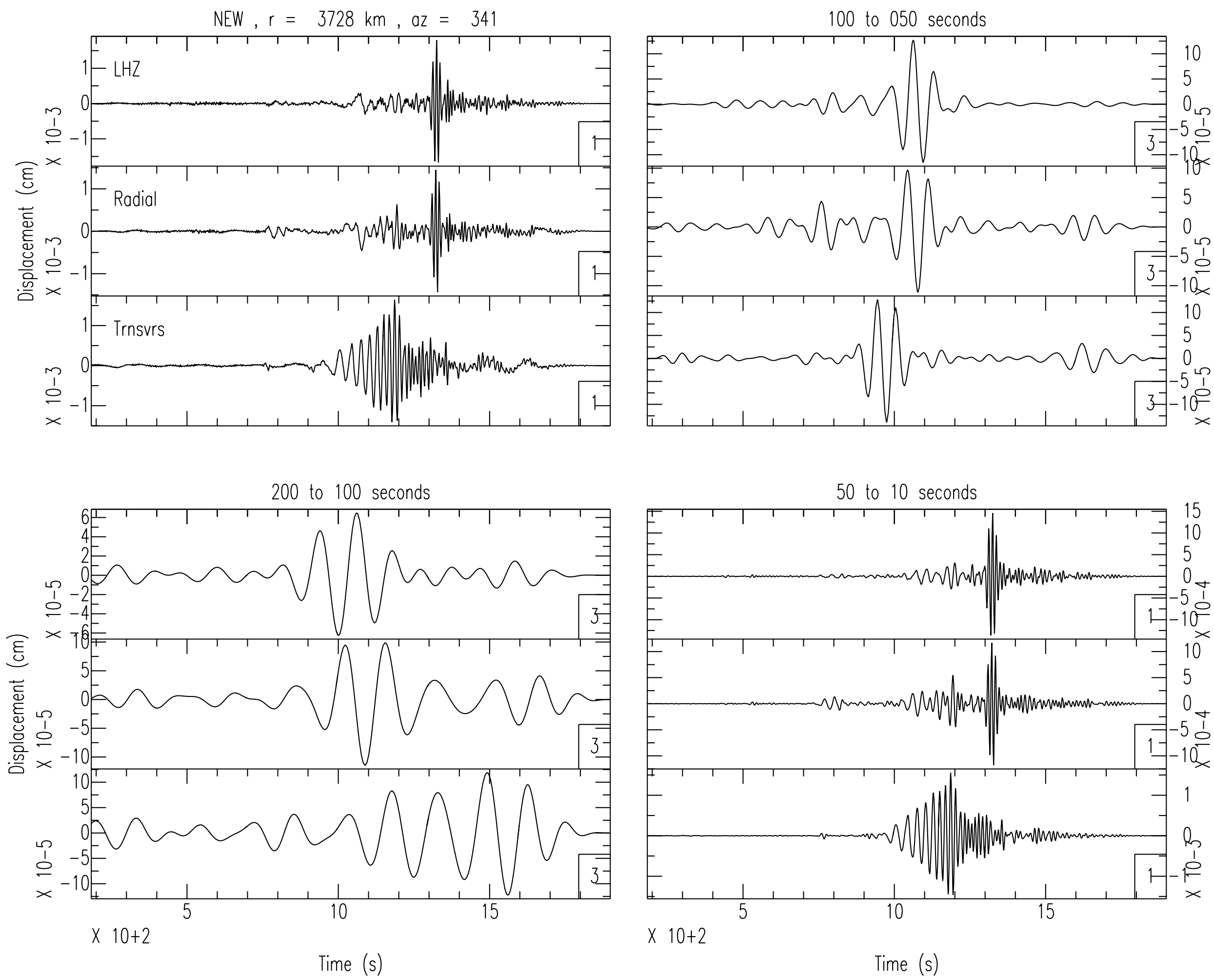


Figure 2–4. Example processing of an earthquake from the Middle America Trench (04/23/96 06:53:35, mb = 5.3) recorded at station NEW, Newport, Washington. The panel in the upper left is unfiltered and the title in that panel lists the distance (r) and the azimuth of the seismometer. The other three panels show the results of band-pass filtering the seismograms using a four-pole, two-pass Butterworth filter into different period ranges. The corner periods of the filters are shown above each panel. The data units are cm, the effects of instruments have been removed.

```
awk '{print $0 " all"}'  tmp >! correct.all.macro
/bin/rm tmp
#
cp ../../correct.macro .
#
sed -e s/event/dsp/ ../Usnsn/plt.all.macro >! plt.all.macro
#
# echo 'Computing displacements'
#
sac <<eof
macro correct.all.macro
quit
eof
#
```

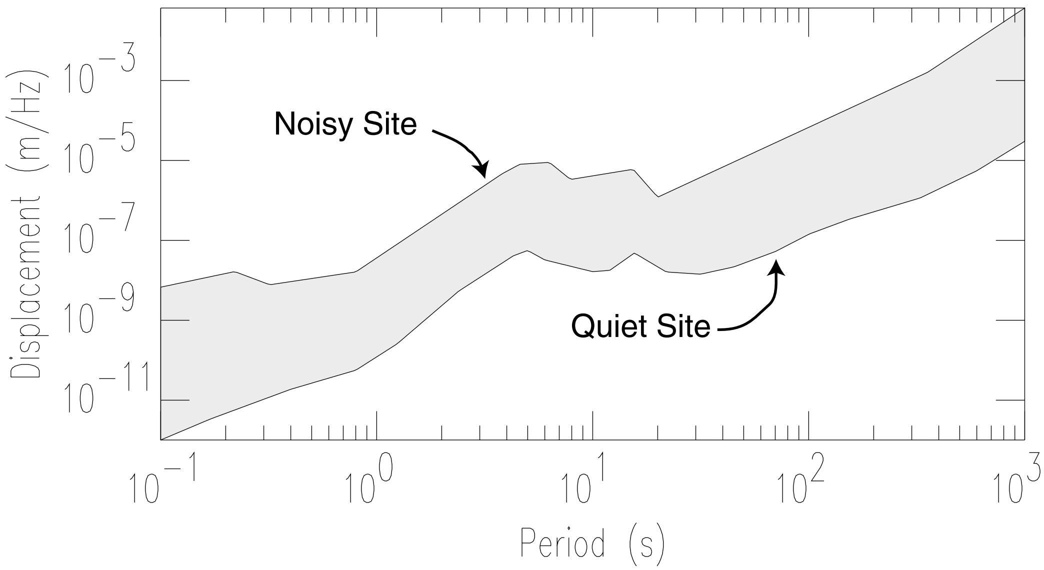


Figure 2–5. Rough guidelines for the bounds of background seismic noise. The actual values vary from site to site and with time.

Seismic Background Noise

Nothing beats experience looking at data for choosing the frequency limits used to window the instrument deconvolution. You can try more “sophisticated” deconvolutions such as time domain approaches, but then you still have to choose a damping or truncation fraction and it is less clear exactly what frequencies you sacrificed for stability. In any event if you were going to use a direct waveform comparison to model the instrument corrected displacements over a broad frequency range you should apply the same cosine taper to the spectra of the synthetic seismograms.

Seismologists study the noise around seismic stations and although the levels vary from place to place, and over time, some reference curves are available. A common reference are the US Geological Survey low and high noise background noise curves(Figure 2–5). We can use these estimates and some simple, classical seismological formulas to estimate the range of observable periods for an earthquake with a given magnitude. The formulas will not work for all data because of the effects of radiation patterns, but the calculations can be useful for examining data and checking the order of magnitude of displacement and spectral amplitudes. We will use the surface-wave magnitude formula

$$M_S = \log A_{20} + 1.66 \log \Delta + 2.0 \qquad (2\text{–}8)$$

Before we can compare the spectral level of noise with the time-domain signal displacement, we have to relate the amplitudes in the time and frequency domain. Following Chapter 10 of Aki and Richards (1980) we can write the relationship between the mean-square time-domain amplitude and the spectral density as

$$\langle f^2(t) \rangle = P(f_0) \cdot 2 \cdot \Delta f \qquad (2\text{–}9)$$

where the brackets indicate that we are dealing with a mean value, and I have assumed that we are looking at a bandwidth, Δf symmetric about the frequency of interest, f_0. If we choose the bandwidth to be equal to one half of f_0, then $\langle f^2(t) \rangle = P(f_0) \cdot f_0$. For the USGS noise curve, we have a level of 1.2×10^{-6} for the high-value at a frequency of 0.05 *Hz* (or a period of 20*s*, where M_S is measured). Thus the mean-square noise level is about

$$1.44 \times 10^{-12} m^2 / Hz \times 0.05 Hz = 7.2 \times 10^{-14} m^2 \qquad (2\text{–}10)$$

or the mean background displacement is approximately $2.7 \times 10^{-7} m$. Assuming a low-noise level of $2 \times 10^{-17} m^2 / Hz$ we find that a quiet site has a background noise level of about $4.5 \times 10^{-9} m$. We can combine the empirical noise and M_S formula to construct a diagram useful for building intuition on the size of surface waves for different magnitudes and distance ranges (Figure 2–6) (don't forget the observed amplitude will be dependent on the depth, faulting geometry and seismometer azimuth).

Ground Noise at Other Periods. To examine the relationships at other periods we need to relate the surface-wave magnitude formula to spectral amplitudes at the other periods. It will help if we first relate M_S to seismic moment, M_0. We can use the relationship (for a 30 bar stress drop)

$$\log M_0 = 1.5 M_S + 16.1 \qquad (2\text{–}11)$$

to estimate the amplitude of a 20 second surface wave as a function of distance and magnitude. To tie those values to another period, we have to assume a source spectral shape. A convenient and simple one is that of Brune (1970, 1971). If we let *f* represent frequency and f_c represent the corner frequency, and M_0 the seismic moment, then the amplitude spectral level, $S(f)$, can be represented as

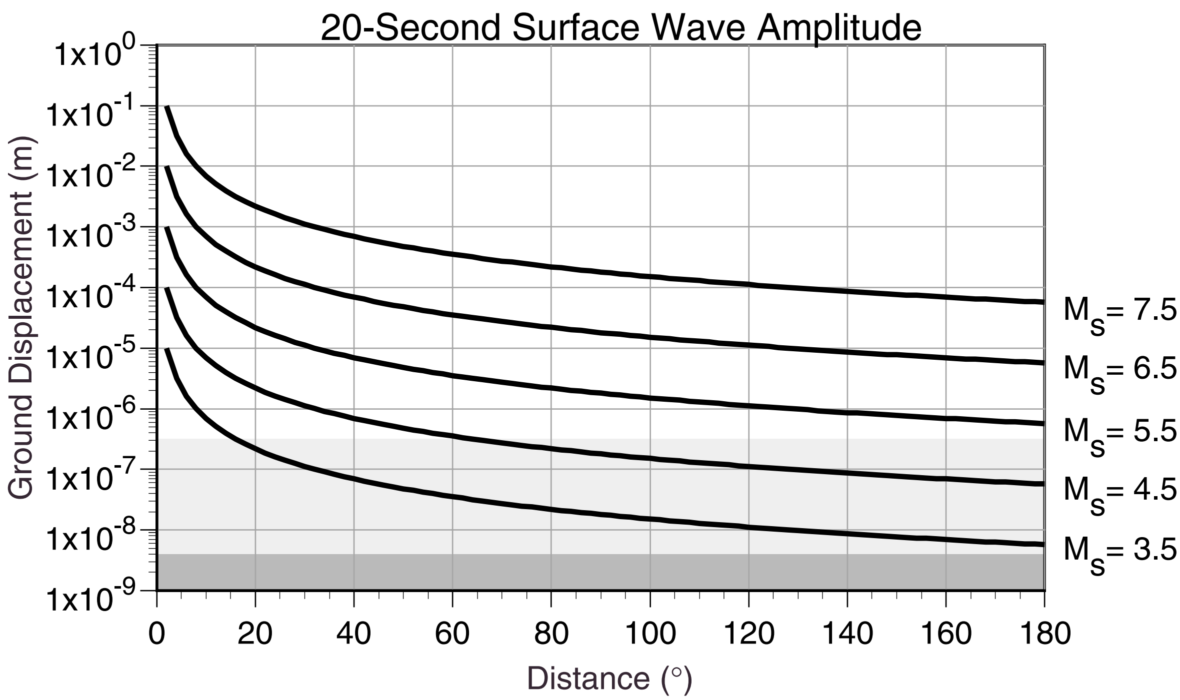


Figure 2–6. The solid curves are the displacements predicted by Equation (2–8) for the magnitudes shown to the right of each curve. The light gray region shows the range of high- to low-noise sites for a period of 20 seconds and a bandwidth equal to one-half the period.

$$S(f) = \frac{M_0}{\left[1 + \left(\frac{f}{f_c}\right)^2\right]} \qquad (2\text{–}12)$$

where the corner frequency, f_c, can be calculated from the expression

$$f_c = 4.9\times10^6 \cdot \beta_0 \cdot \left(\frac{\Delta\sigma}{M_0}\right)^{1/3} \qquad (2\text{–}13)$$

where β_0 is the shear velocity in *km/s* near the source, $\Delta\sigma$ is the stress drop in bars, and the moment, M_0 is in *dyne-cm*. You can calculate the 20s Rayleigh wave amplitude using the formula for M_S and then compute a scale factor for the amplitude at another period by assuming the source spectral shape given in Equation (2–13). This is not strictly true because Ms is a time-domain measurement and the relationship between the time and frequency domain amplitudes is not quite so simple, you must account for a range of spectral amplitudes to get the time-domain amplitude.

3 • Estimating Group Velocities

An Overview

To perform a multiple-filter analysis of seismograms, I have written an interactive program called "*pgswmfa*" (graphics are executed using the *PGPLOT* graphics library, see the appendix that describes the package.). The idea behind multiple-filter analysis is simple. Dispersed surface waves arrive at the seismic station at a time that depends on the wave frequency (or period). To estimate the group arrival time as a function of frequency the original seismogram is filtered using narrow-band Gaussian filters and the arrival time of the peak of the filtered signal envelope is used as an estimate of the group-delay time. Assuming that the wave followed the great-circle arc between the source and the receiver we can compute the group velocity from the arrival time. I'll begin with a description of how to use the program and then follow up with details on the calculations being performed. As with most complicated processing, some specific algorithms are necessary to successfully implement the general scheme.

Environment Variables. You should set one to three environment variables before you start the program. The first, *PGPLOT_DIR*, is required and should be set to the contain the directory that contains the *PGPLOT* library. To set an environment variable, add

```
setenv PGPLOT_DIR /geo/PGPLOT/bin
```

to your *.cshrc* or *.login* file. The other environment variables provide the location of the reference curves that you can optionally have plotted for comparison with the estimates from this program. They are called *PREM_R_GVEL* and *PREM_L_GVEL* and they point to the Rayleigh and Love wave reference curves. These files are two-column ASCII format containing one period and group velocity on each line. The maximum number of values in the reference curve is 100.

Setting the SAC Header Values. Since we are going to estimate absolute travel times between the source and receiver, the distance and origin time of the signal must be stored in the SAC file header. It is helpful for record keeping to store as much information in the header as you can. If you don't store many values you may see some error messages when you save a dispersion file. These errors are not fatal but occur because I tried to keep relevant information in the output file. The SAC values that are helpful and that are stored with the dispersion measurements are the event latitude, longitude, and depth and the station latitude and longitude and the seismometer component name. Obviously if you are processing signals for the purpose of estimating the source depth, you won't know that at the time you isolate the surface wave.

Preparing The Input Files

"list" and "in.swmfa". If you are planning to process a substantial number of files you should create two files before you launch the program. First, create a file called "*list*" that contains the list of files (one seismogram file name per line) that you want to process. This saves you from typing the names of each file during the processing.

Several other parameters are also usually common to signals from a particular event or project and these can be stored in a file called "*in.swmfa*" (Table 3-14 on page 18). Again, this is to save time entering the same numbers over-and-over while you process a collection of waveforms. Parameter values stored in the file *in.swmfa* are parsed using free-format. The first line should contain the Gaussian filter width parameter, the second line should contain the minimum and maximum group velocities, the third line should contain the minimum and maximum periods, and the final line should contain the number of periods to use. The Gaussian width parameter is not the default for the program, a distance- and period-dependent value is the default (this is discussed in detail later). An example *in.swmfa* file is listed in the second column of Table 3-14.

Table 3-14: Sample file *in.swmfa*

Line		Comments
1	57	Gaussian filter width parameter
2	2.25,4.75	Minimum and maximum group velocities
3	10,100	Minimum and maximum periods
4	20	The number of periods to sample

The periods are sampled with a uniform, decimal logarithmic sample interval but the velocities are sampled linearly. Choose values that are reasonable for the seismograms you are processing. For example, it's a waste of time looking at periods of 200 seconds in signals from a magnitude 4.8 earthquake.

Seismogram File-name Conventions. Another time-saving feature is the automatic association of files that end in "z" with vertical components and Rayleigh waves, and files that end with "t" with transverse components and Love waves.

Program Execution and Output

When you start the program it looks for the files "*list*" and "*in.swmfa*" (it will prompt for the input if it doesn't find them). Once that information has been parsed, the filters are applied to the data and a contour image of the resulting envelopes as a function of period and group velocity is contructed. Initial estimates of the group velocities are computed using the maximum amplitude at each period and a spline with two "knots" is fit to the group velocity values. A graphics window is opened and the multiple-filter spectrum, the seismogram as a function of group velocity, and the amplitude spectrum are plotted in three panels. Sampled points are shown as diamonds on the spectra, and the reference values are shown as triangles on the contour plot. The spline-fit through the observations is shown as a solid curve (Figure 3–7).

Once the graphics are displayed, a character-key based menu (Figure 3–8) is listed in the "terminal" window and the cursor over the image becomes "active" and the program enters an event loop and waits for action from the user. As you work with the program, there are two places where you will enter information. Initially, the "hot" area is the image in the graphics window. For example, to see the numerical values of the current group velocity estimates position the cursor over the image and type the letter "v". A list of values appears in the "terminal" window from which you launched the program. Some options require a response typed in the terminal window. For example, if you decide to increase the number of knots in the cubic-spline fit you type a "k" with the cursor over the image, and then supply the new number of knots in the terminal window. A description of the menu options is provided in Figure 3–8. An example of illustrating the result of placing the cursor over the image and using the "h" for high-pass filter is shown in Figure 3–9.

Noise Spectra

To help guide interpretation of the signal, a plot of the spectral amplitudes is shown on the far right of the main graphics window. The red diamonds identify the spectral amplitudes at the filter center periods, the solid line shows the more finely sampled spectrum. The spectrum is a simple FFT amplitude spectrum and is meant to be used a quick-and-dirty view of the spectral amplitudes. Information on the noise is available from two sources. The default option simply shows the high- and low-noise USGS ground-noise spectra which can be used a crude guide to the general shape of seismic ground noise. Preferably, if you are fortunate to have long time series then you can use the signal before the P-wave to compute a spectral amplitude of the noise. To use this

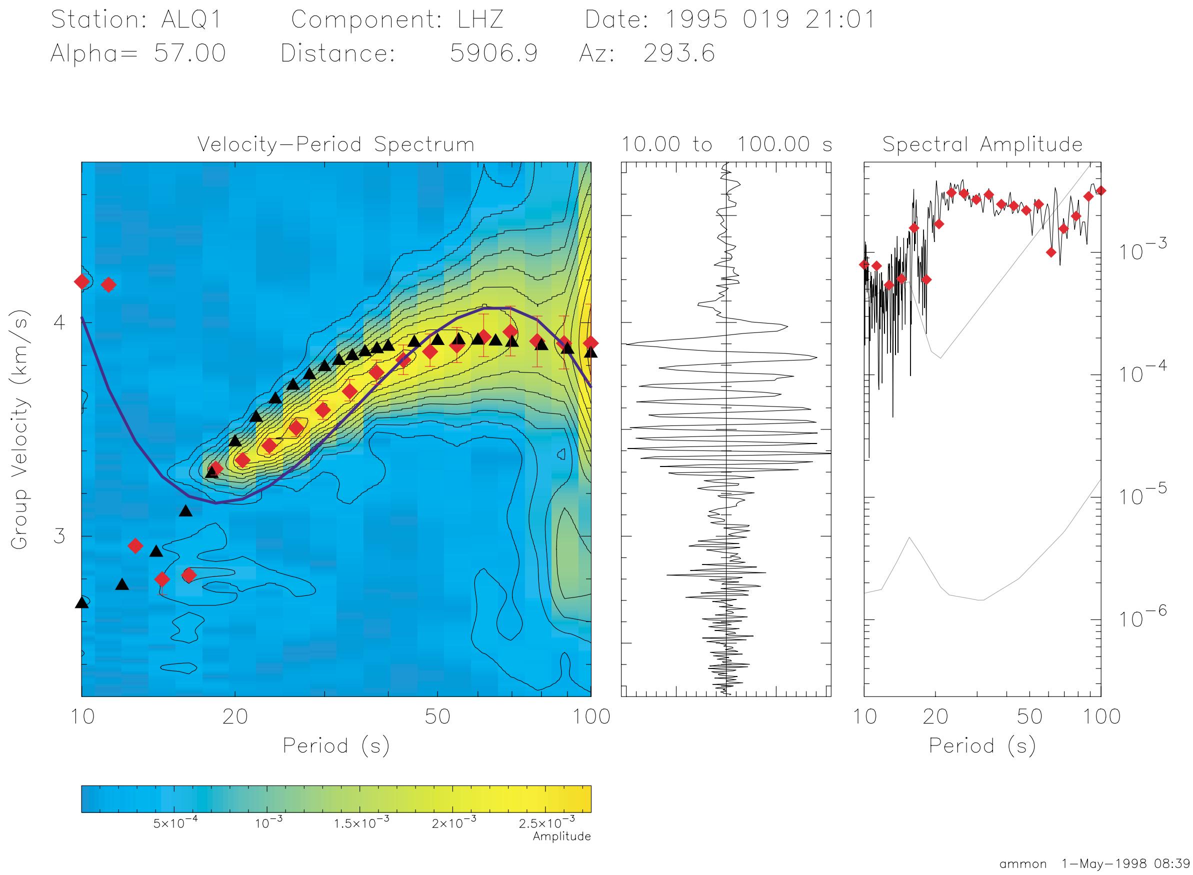


Figure 3–7. Display from program "pgswmfa". On the left is a contour diagram of the seismogram envelope. The current group velocity points are identified by diamonds, the spline by the blue line, and the reference curve by the triangles. In the middle, the seismogram is displayed along the group velocity axis. On the right the amplitude spectrum and the sampled points (shown by diamonds) are shown along with the USGS low- and high-noise curves. The values below a period of about 18 seconds are lost in the noise and substantially corrupt the spline fit.

option you must start the program with the "*-n*" command line flag (*pgswmfa -n*) and you must have the P-wave time marked in the SAC files (stored in the "*a*" header field). If you don't have the header value set in all of your files, you can still use the "*-n*" option but for those stations with no "*a*" marker you will see a SAC *getfhv* error, which you can safely ignore. The program will show the USGS curves when the header is not set. The noise spectrum if also a quick, simple computation. The noise is cut from the seismogram, the mean is removed, the signal is windowed with a Bartlett (triangle) taper, the FFT and amplitude spectrum are computed. The logarithm of the amplitudes are smoothed using two passes of a three-point averaging triangle (1,2,1) to produce a simple spectral curve. If you see something strange, check the number of points and the duration of the signal used to compute the noise spectrum (the values are listed to the command terminal). A sample output is shown in Figure 3–10. The example also illustrates how a spectral

```
*********************************************
 MENU:
 m = list this menu
 r = refresh the plot
 l = truncate all periods lower
 h = truncate all periods higher
 u = use all periods (undoes l or h command)
 d = remove value at cursor position
 i = set velocity to cursor location
 w = write the dispersion
 n = go on to next waveform
 k = change number of knots in spline fit
 v = list the values to the terminal
 p = save postscript file
 c = clean with mode-isolation filter
 b = batch clean with mode-isolation filter
 x = clean with mode-isolation filter
     using the reference curve
 y = batch clean with mode-isolation filter
     (using the reference curve)
 g = change Gaussian width parameter
 s = save the seismogram to a file
 a = adjust the color range
 z = adjust the image scaling factor
 e = examine signals for a specific period
 o = isolate and remove mode to look at overtones
 q = quit
*********************************************
 Command-line arguments:
 -b invokes batch operation
 -f defaults to fixed Gaussian width parameter
    which is read in from in.swmfa
 -n Use the signal before the SAC "A" marker
    to estimate a noise spectrum
*********************************************
```

Figure 3–8. Command menu for pgswmfa. The interface is simple by modern GUI standards, but streamlined for rapid use of the program. Note that some of the commands only work with lower case characters (x, d, a).

notch and multi-pathing appear on a velocity spectrum image. The spline fit allows you to interpolate directly through the notch, but you must be careful using such data – such parameters as instantaneous period can be less stable in regions with low signal amplitude.

Output Files

The specific output depends on the options that you choose. For example you can output a postscript plot, a mode-isolated seismogram, the group velocities, or nothing at all. The seismogram output is in SAC format, and the header will be identical to the input file, so all absolute time information is retained. The group velocities are output in two formats: a SAC file (with event and station latitude and longitude, etc. information) and an ASCII format. The XY format SAC file contains the splined group velocities as a function of instantaneous period. The ASCII file includes the spectral amplitudes, the splined and unsplined values, the Gaussian-filter center and the instantaneous periods, estimated of the range of group velocities estimated by measuring the width of the group-velocity ridge on the image (95% of the peak value), and the real and imagi-

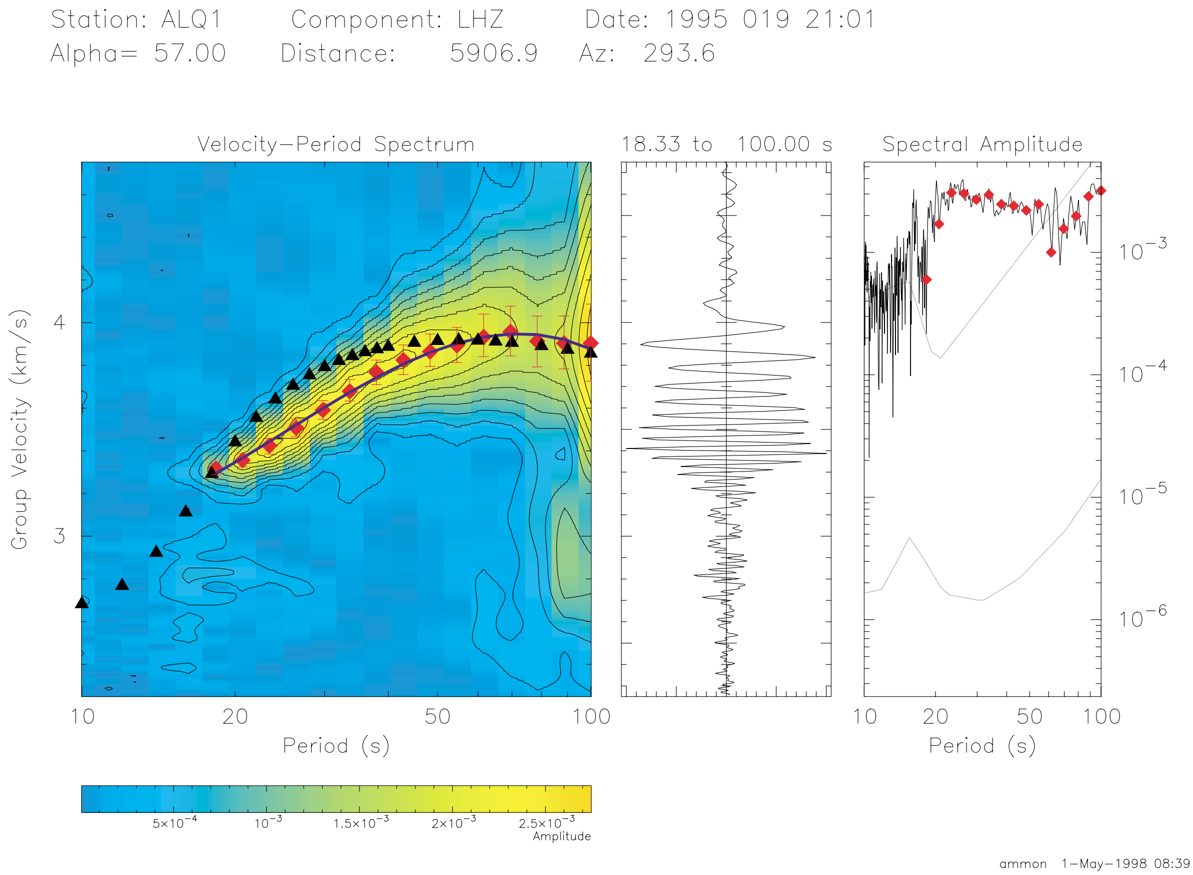


Figure 3–9. Display from program "pswmfa". Same format as previous Figure 3–7. The short-period measurements have been filtered away using the "l" option and the spline closely follows the peak amplitudes. The seismogram has been filtered using a simple box-car filter to reflect the change in period range (this does not affect the group velocity estimates). The gray noise curves in the spectral plot identify standard USGS low- and high-noise values.

nary parts of the spectrum sampled at the periods nearest to the center period. Note that the measured group velocity may not have a period close to the filter center period, but the amplitude points are sampled from the seismogram spectrum at the period corresponding to the filter center period. The ASCII file format consists of a header followed by the measurements. Each line in the header begins with a pound-sign and contains information on the program, station, event, processor, input file location, and processing parameters. Interpretation of most of the values is obvious in context; the mode isolation flag is zero if no mode isolation was not performed, and unity if it was. An example is shown in Figure 3–11.

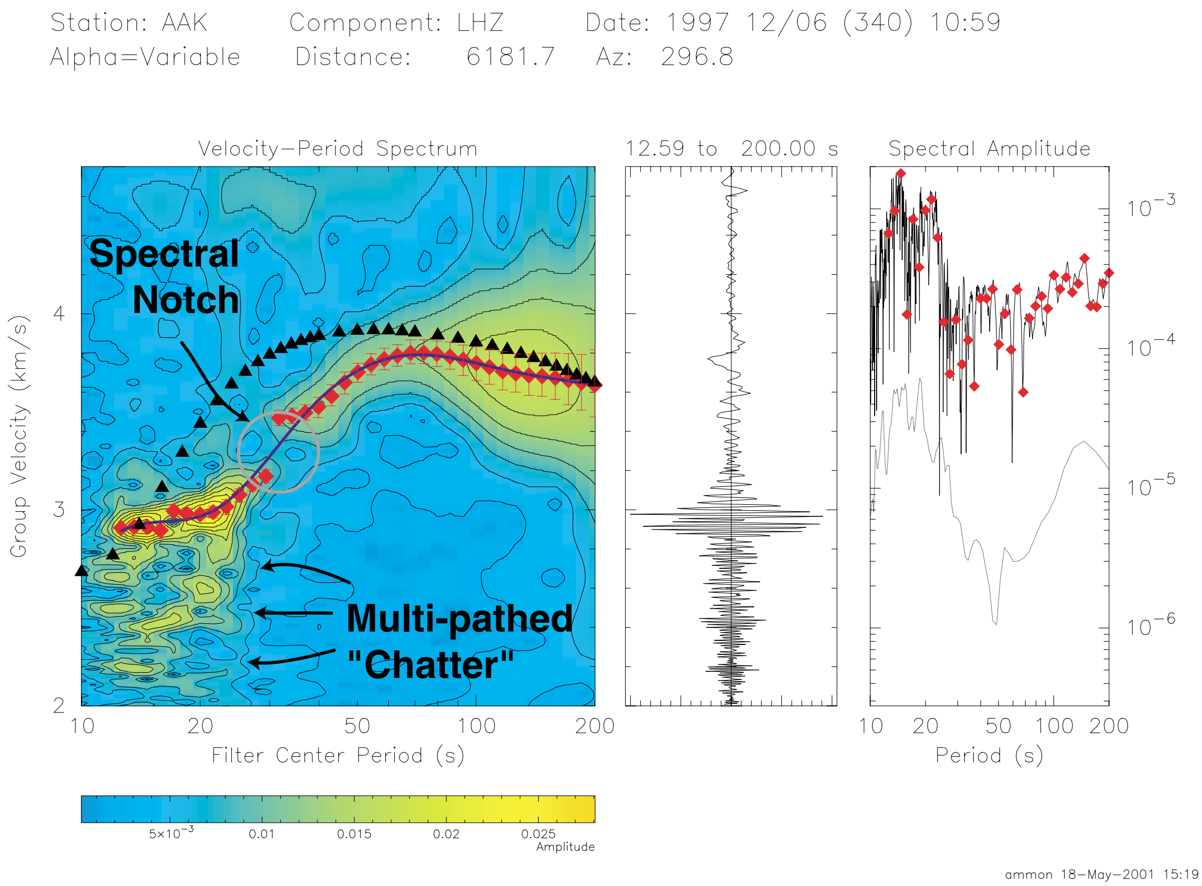


Figure 3–10. Multiple filter analysis window with noise estimated using the signal in front of the P-wave. The gray line in the spectral plot on the right shows the estimated noise spectral amplitude, which is below the signal amplitude for all periods of interest. Note that the shape of the noise curve impacts the shape of the surface-wave spectrum. Noise is often low in the 30-60 seconds period range.

Mode Isolation

An option in the processing is the use a mode isolation filter to extract the surface-wave signal from the original record. The procedure is simple: A unit-amplitude signal with a phase estimated from the group velocities is constructed and cross-correlated with the original signal. The result of this cross-correlation is that any component of the original signal with a phase matching the filter is compressed to a wavelet near zero phase in the cross-correlation. If we window the cross-correlation to attenuate any signals far from the zero-phase and then apply the complex conjugate of the original filter, we have isolated the signals from the original that have group delays near the

```
# SWMFA Version 2.1.0
# Station Info: ALQ1     34.9425   -106.4575  LHZ
# Event Info:   28.9520    -43.3400  10.
# User and Time Processed: ammon        Wed Apr 29 08:31
# File name: ALQ_dsp.z
#  Directory
#   /home/mantle/ammon/Programs/SWprocessing/pgSWMFA_Spline/TEST01
# Filter Width Parameter: C 57.00  Mode Isolation: 0  Peak Error Fraction: 0.95
# Period Range Processed:   10.00   100.00
# Group Velocity Range Processed:    2.25     4.75
#
#   T0    InstT    spV     pkV     ldV     hdV       Real(A)        Imag(A)
  18.33   19.26   3.294   3.317   0.030   0.022     0.326478E+01    0.739084E+01
  20.69   21.21   3.367   3.356   0.030   0.031     0.719352E+01   -0.206174E+02
  23.36   23.73   3.443   3.424   0.039   0.032    -0.368977E+02    0.226225E+02
  26.37   26.31   3.521   3.507   0.041   0.042     0.378375E+02    0.287128E+02
  29.76   29.57   3.599   3.591   0.043   0.044     0.132940E+02   -0.326129E+02
  33.60   33.21   3.674   3.679   0.045   0.056    -0.423533E+02    0.708957E+01
  37.93   37.70   3.743   3.767   0.057   0.059     0.180172E+02    0.202651E+02
  42.81   42.65   3.806   3.825   0.068   0.071    -0.252442E+02   -0.141792E+02
  48.33   48.08   3.860   3.864   0.070   0.083    -0.264734E+02    0.504105E+00
  54.56   54.37   3.903   3.894   0.080   0.084     0.124932E+02    0.168034E+02
  61.58   61.25   3.933   3.933   0.092   0.108     0.411962E+01   -0.151241E+02
  69.52   69.08   3.947   3.957   0.113   0.109     0.233636E+02   -0.106671E+02
  78.48   77.98   3.944   3.913   0.111   0.118     0.959448E+01   -0.185010E+02
  88.59   91.01   3.921   3.904   0.120   0.128    -0.208462E+02   -0.586906E+01
 100.00  101.42   3.877   3.904   0.177   0.184    -0.178174E+00    0.495954E+02
```

Figure 3–11. Sample output file from surface-wave multiple filter analysis code.

estimate of the group velocities we used to construct the isolation filter. The isolation filter, $C(\omega)$ is constructed using the group velocities

$$C(\omega) = e^{i\theta(\omega)} \tag{3–15}$$

where $\theta(\omega)$ is the integral of the group delay

$$\theta(\omega) = \int_0^{\omega} t_g(\omega) d\omega \tag{3–16}$$

I compute the integral using a simple trapezoidal rule and set the group velocity outside the range of selected periods to be infinite. An example is shown in Figure 3–12.

The cross-correlation window can be varied interactively during the filtering process and you are allowed to change the width of the window and the fraction of the window that is constructed of a cosine taper. For example a fraction of 0.5 produces a simple cosine taper, smaller fractions produce a hybrid window that is flat between the cosine-tapered edges. When you are performing the

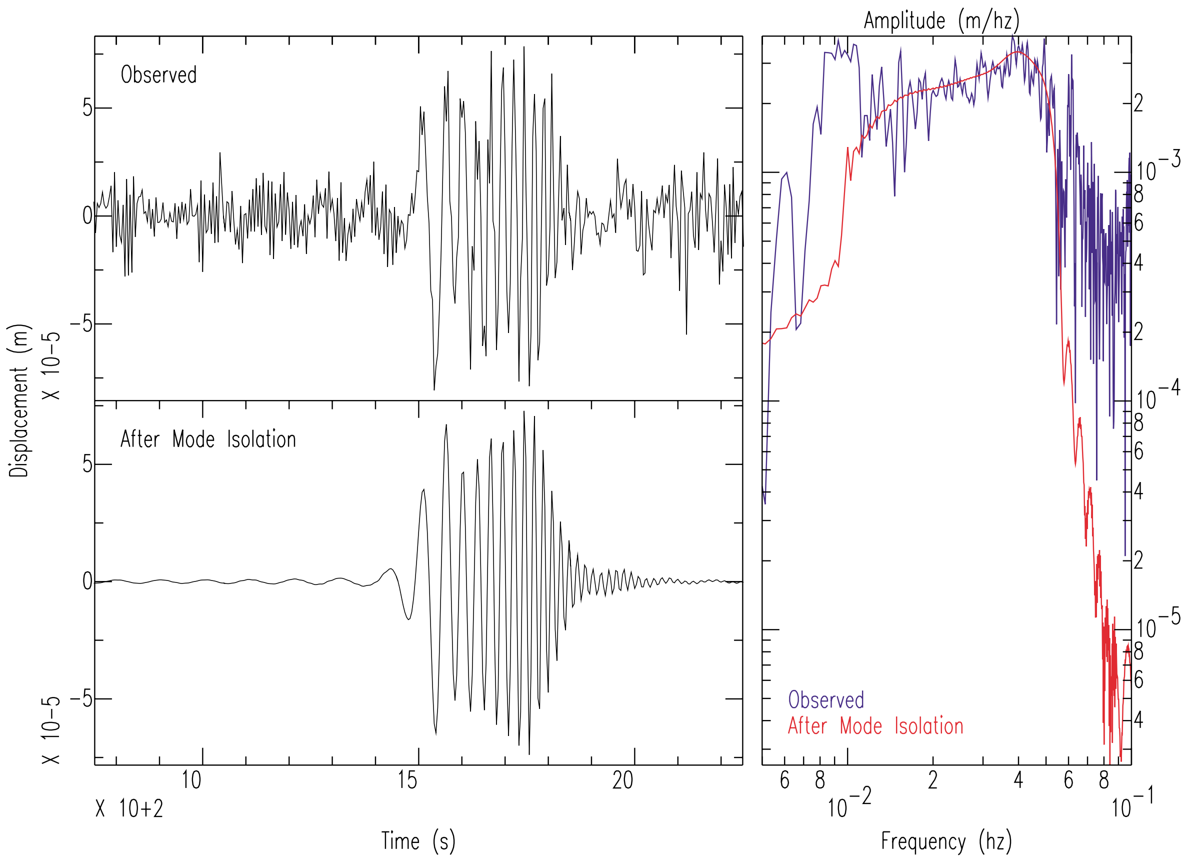


Figure 3–12. The left panel is a plot of the original signal (top) and the mode-isolation filtered time series. Mode isolation extracts the part of the signal with a group delay similar to that described by the group velocity curve. On the right is shown the spectra of the original and isolation-filtered signals. Windowing of the cross-correlation function greatly smooths the spectrum.

filtering, the window, the original, and the windowed signal are shown in the upper panel. The lower panel contains the original and isolation-filtered seismograms. One affect of the mode isolation is a smooth, stable estimate of the spectral amplitudes. The time-domain windowing procedure is equivalent to a convolution (smoothing) in the frequency domain. The isolated mode's spectrum is much smoother than the original seismogram spectrum and thus much more suitable for source studies.

Mode Isolation Using a Reference Curve

One common application of multiple filter analysis is the isolation of the surface-wave signal from background or signal generated seismic noise. Although *pgswmfa* is designed to estimate group velocities and use them to isolate the mode, it may be that you are looking a small-event signals traveling along a path calibrated using a larger event. In that case you might want to use a master group velocity curve for the path to isolate the surface-waves. The interactive options “x” and "y" allow you to accomplish this task. The procedure is straight forward but you must set up the reference curves beforehand. An example may make the procedure clear and illustrate some problems that blindly using this approach can cause. We'll look again at the seismogram used earlier in Figure 3–10. We now perform a mode-isolation filter using the PREM reference values (dark triangles in Figure 3–10) and with a group velocity curve estimated using the observed signal. The results are shown in Figure 3–13. I have two points to make with the example. First, you can use a reference curve to perform mode isolation; second, be sure that the curve is appropriate for the observations.

Overtone Isolation

The program has the capability to isolate the higher modes. Before you begin, make sure that your group velocity window is large enough to encompass the higher modes. Then work as normal to produce the group velocity curve for the fundamental mode. Once you have that mode's group velocity curve we can use that to isolate and subtract the fundamental mode from the seismogram. Then the computation resumes with a multiple-filter analysis on the residual seismogram. The procedure works relatively well, but you should clearly separate the high-mode outputs from the fundamental mode output files. I only write out one SAC file with a particular name, but the ASCII output files for the generic (fundamental mode) and the overtone processed files differ in two ways. First, I added one flag to the output file to distinguish between the types of measurements. An extra header line containing

```
# Overtone
```

will appear in files where you use the "*o*" option. Second, the default (fundamental mode) ASCII files end with a suffix ".ga". Signals processed with the overtone option end with the suffix ".go". An example of the processing steps is shown in the next section. Although you can make overtone measurements with *pgswmfa*, I recommend that you use a more sophisticated approach exploiting a true mode-isolation filter (a seismogram containing only the mode that you want). Otherwise it can be hard to identify the appropriate mode.

More Examples

Some of the ideas described in the preceding sections are illustrated in the following examples. First we look at a comparison of the vertical and transverse component spectra for a thrust faulting event in Kamchatka, recorded at station KIV near the Caspian Sea (Figure 3–14). These waves

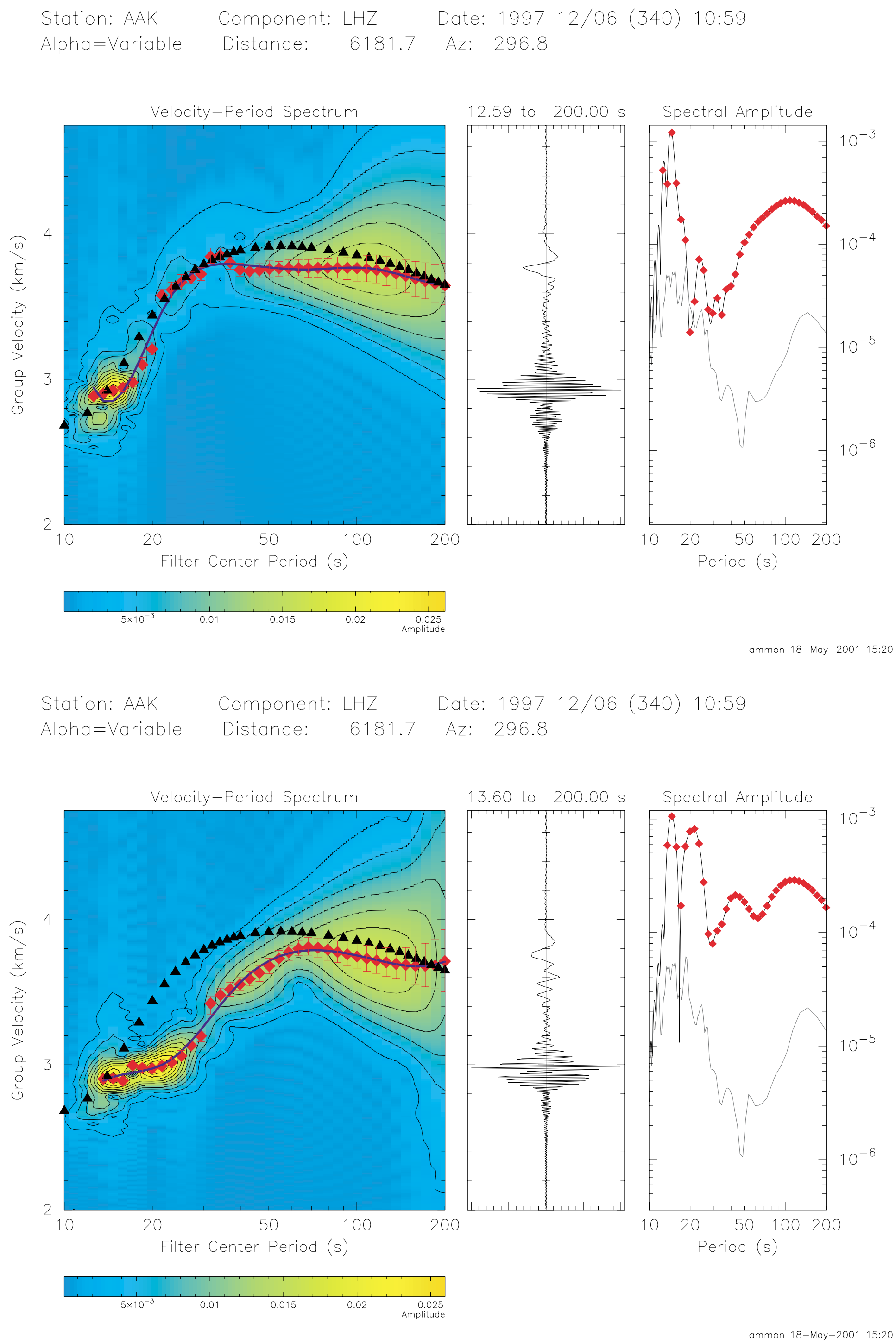


Figure 3–13. Comparison of mode-isolation using PREM and using the observed group velocities on a 6000 km long path. PREM is inappropriate in the intermediate period range (20-50 s) and actually filters out some of these signals, and produces an incorrect spectrum and incorrect group velocities.

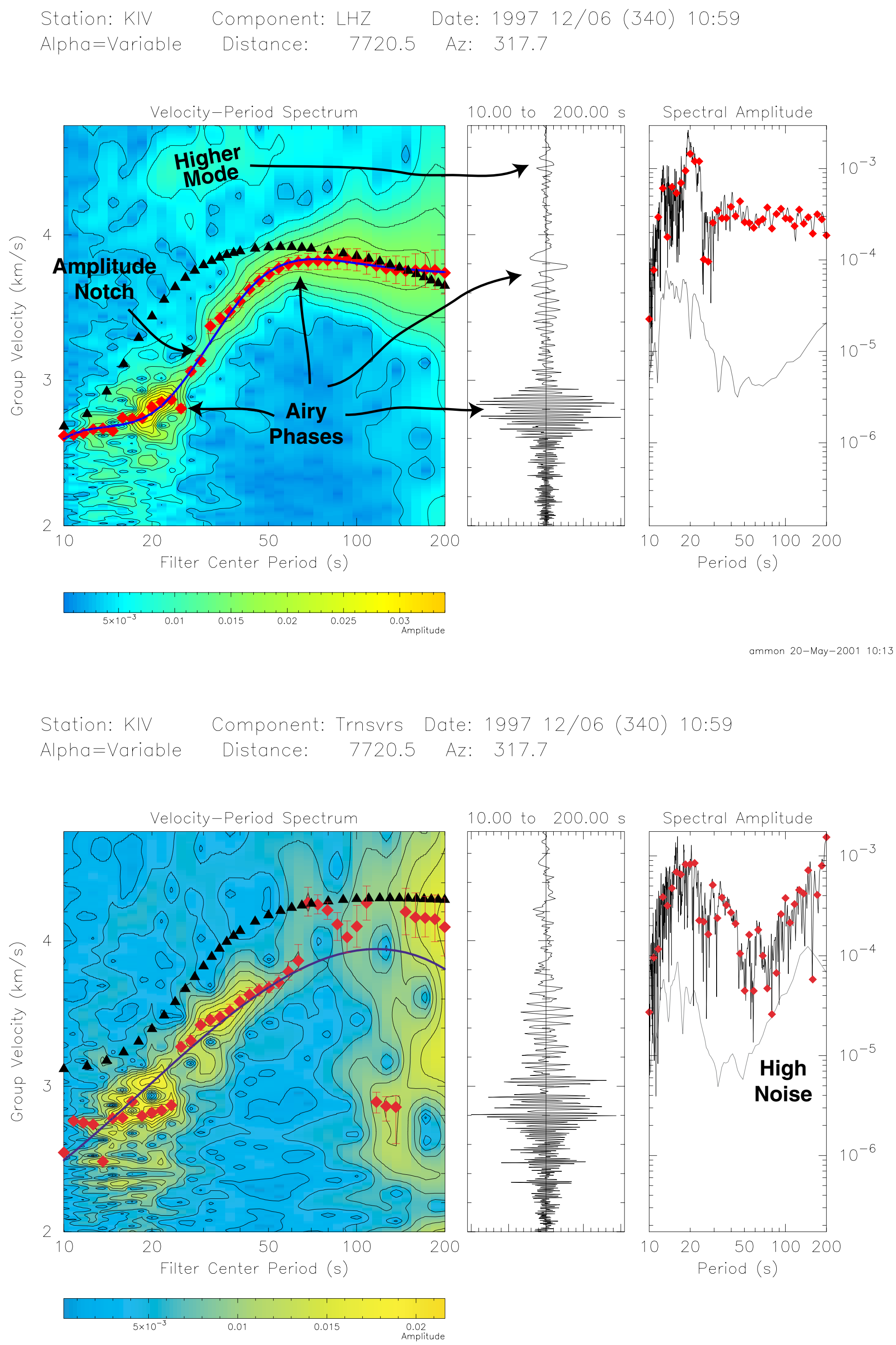


Figure 3–14. Sample processing to vertical and transverse components for a thrust event near Kamchatka. See text for discussion.

traveled about 60° across Eurasia from the source to the station. The vertical component has a nice Rayleigh wave observations but the transverse contains a relatively small amplitude Love wave recorded on a channel with a noise level approximately 10 times higher at the long periods. Little useful information is available on the transverse, but the vertical signal allows a stable estimate of the group velocity between the periods of 20 to 200 s period. I've identified several features in the signal on the velocity spectrum image and the corresponding signals on the seismogram. One reason for showing three panels on the graphics window is to allow you to see the same features in three different "domains". The three different views contain the same data and by examining the results in each panel you should be able to understand complexities in the group velocity curves.

Signals from the same event provide another example of the affects of phase-match or mode-isolation filtering the signal. The before-and-after presentations are contained in Figure 3–15. Again we can see two Airy phases in the signal. The mode-isolation filter did a superb job of cleaning up the seismogram and smoothing the amplitude spectrum. The signal-to-noise ratio suggests that most measurements in the range from just below 20 s to 200 s are reliable. We can also process this signals to isolate overtones. Starting with the same signal shown in the top panels of Figure 3–15, we first set the group velocity curve to match the fundamental mode (as we normally do). Then we execute the "o" option of pgswmfa, which isolates the fundamental mode and subtracts it from the original signal, leaving behind mulitpathed surface waves and higher modes. The results are shown in Figure 3–16 and Figure 3–17. In this case the overtone dispersion is nicely isolated and you can see the dispersion directly in the waveform.

Summary

The multiple filter analysis procedure so far can be summarized as

1. Unpack the data and set up the directory structure
2. Examine the data (*pltevent.macro*)
3. Remove the instrument and window the signal (*correct.macro*)
4. Prepare the input files for *pgswmfa*.
5. Run *pgswmfa* to estimate the bandwidth and group velocities.

An example that illustrates the various output values possible is shown in Figure 3–18. For this example the effects of instantaneous period and spline fits appear minor, but the isolation-filtered signal actually has an approximately seven-second difference between the filter center period and

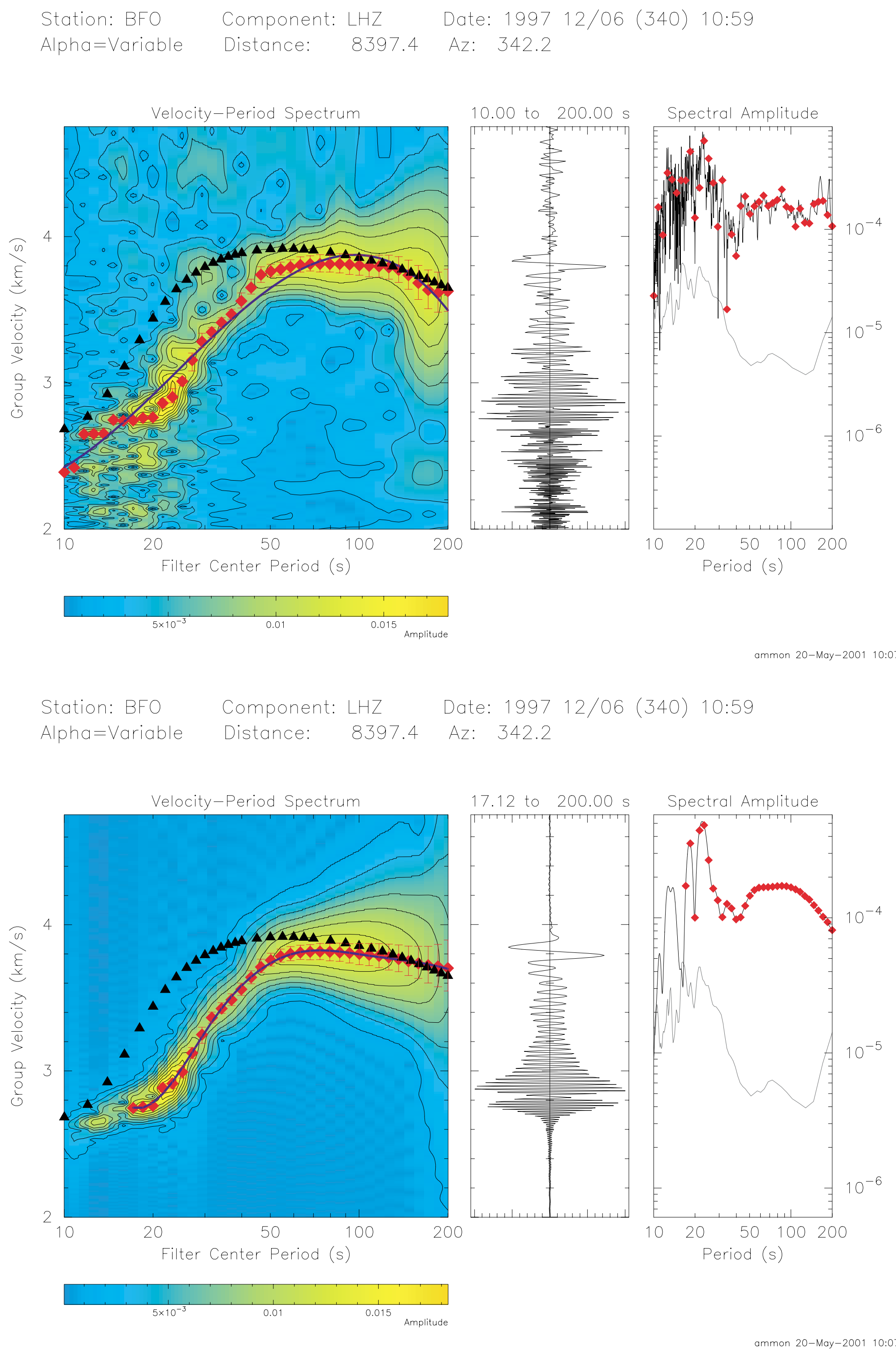


Figure 3–15. Processing example for Rayleigh wave from Kamchatka event. The original signal is shown on the top, the signal after mode-isolation is shown on the bottom. See text for discussion.

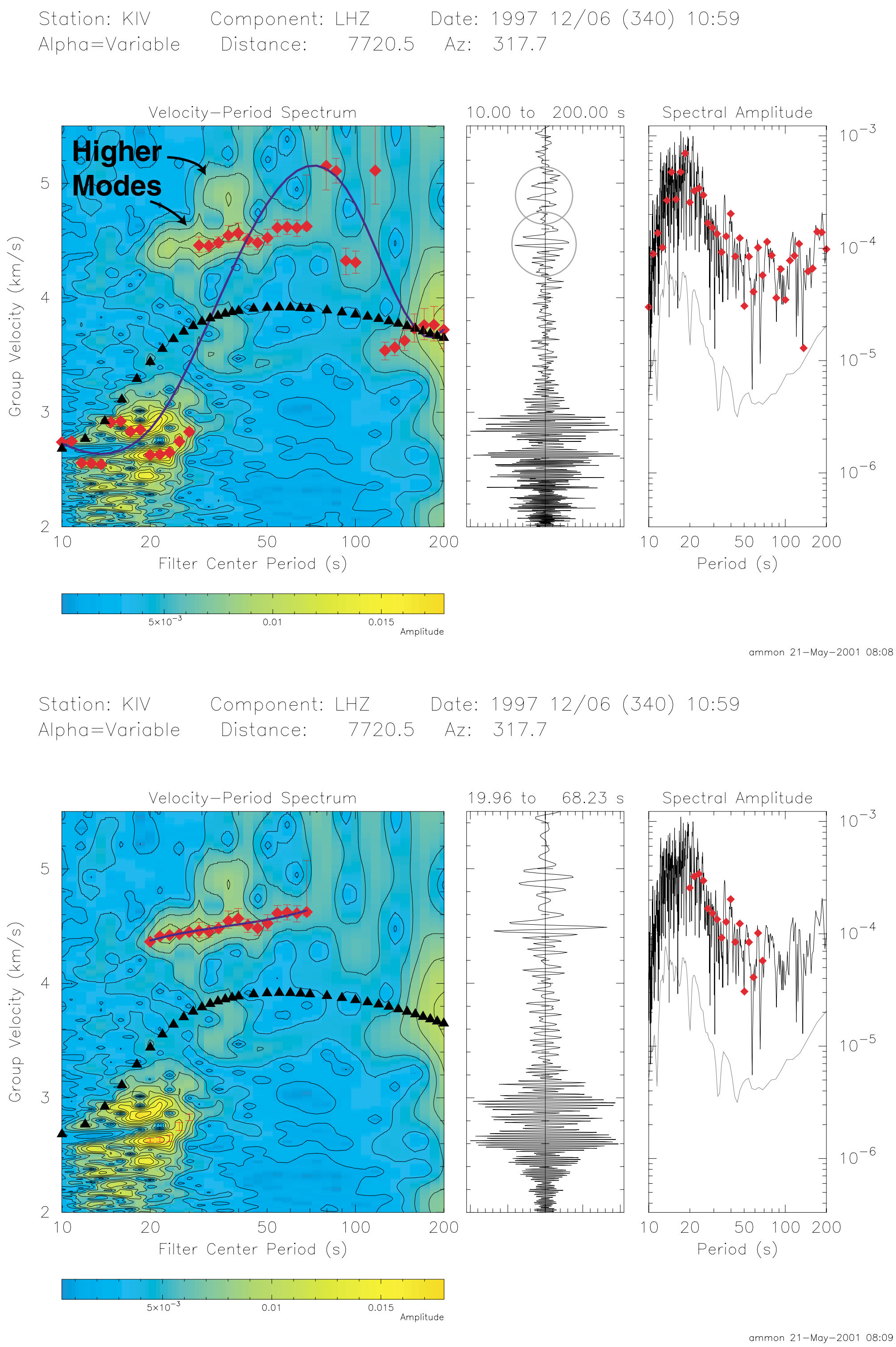
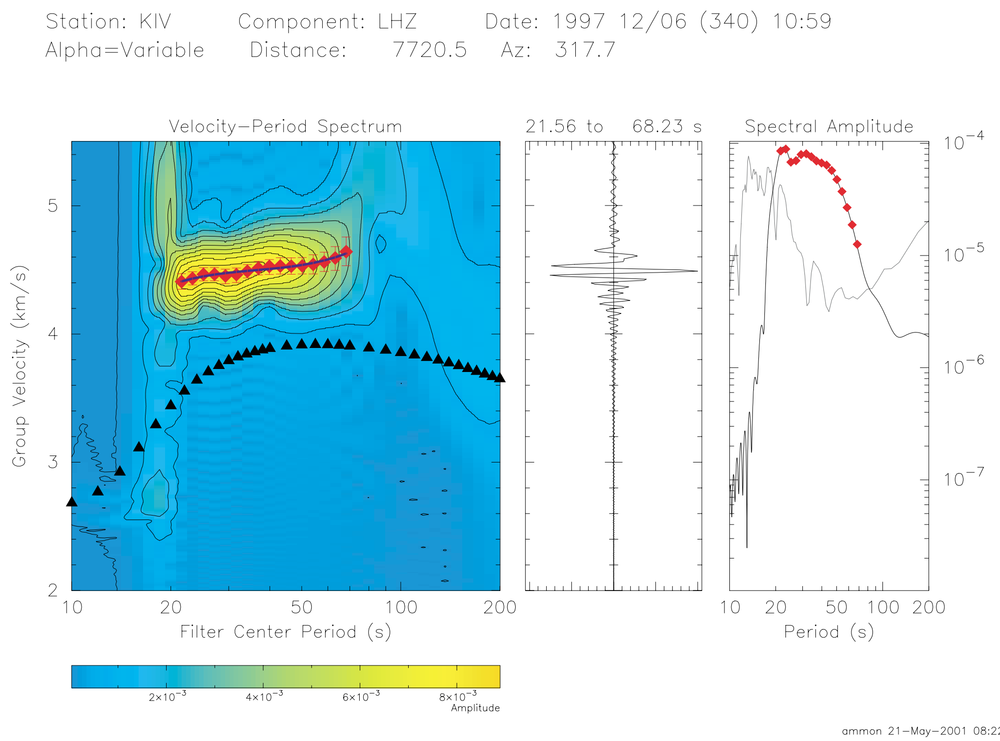


Figure 3–16. Overtone isolation example. The original signal is shown in Figure 3–15. The top panel shows the result of removing the fundamental mode with the “o” command. Two higher modes are identified on the image and seismogram. The lower panel is the result of filtering and using the interpolation command to track the later arriving overtone.




```
# SWMFA Version 3.9.0
# Station Info: KIV      43.9553     42.6863   LHZ
# Event Info:    53.9720    161.9090   33.
# User and Time Processed: ammon        Mon May 21 08:21
# File name: kiv_d_070_a_318_ldsp.z
#  Directory
#   /users/ammon/Programs/SWprocessing/pgSWMFA_Spline/TEST_KamE01/KIV
# Filter Width Parameter: V -99.00  Mode Isolation: 1  Peak Error Fraction: 0.95
# Period Range Processed:    10.00   200.00
# Group Velocity Range Processed:    2.00     5.50
# Overtone
#
#    T0     InstT    spV     pkV     ldV     hdV        Real(A)          Imag(A)
  21.56   22.00   4.411   4.409   0.060   0.061    0.183479E+01   -0.206527E+01
  23.28   23.20   4.433   4.434   0.050   0.052   -0.130711E+01   -0.250571E+01
  25.14   25.02   4.450   4.466   0.051   0.052    0.759799E+00    0.202590E+01
  27.14   27.21   4.464   4.467   0.051   0.052    0.107830E+01    0.198670E+01
  29.31   29.42   4.475   4.454   0.061   0.052   -0.253093E+01   -0.328136E+00
  31.65   31.60   4.485   4.463   0.061   0.052    0.232865E+01    0.111868E+01
  34.18   33.96   4.493   4.489   0.052   0.053    0.216173E+01    0.109426E+01
  36.91   36.59   4.500   4.513   0.052   0.053   -0.104138E+01    0.196505E+01
  39.85   39.42   4.507   4.527   0.063   0.054    0.212800E+01    0.198364E+00
  43.04   42.32   4.516   4.531   0.063   0.065    0.168861E+01   -0.114381E+01
  46.47   45.31   4.526   4.529   0.063   0.065    0.181473E+01   -0.499021E-01
  50.18   48.41   4.539   4.532   0.063   0.065   -0.386593E+00    0.144654E+01
  54.19   51.63   4.554   4.542   0.074   0.076    0.361710E+00   -0.110528E+01
  58.52   54.89   4.574   4.562   0.074   0.077   -0.369635E+00    0.744632E+00
  63.19   58.32   4.598   4.593   0.096   0.089   -0.107279E+00    0.567371E+00
  68.23   61.66   4.628   4.642   0.109   0.114   -0.375713E+00    0.788473E-01
```

Figure 3–17. Results of mode-isolation filtering the overtone and the associated output file.

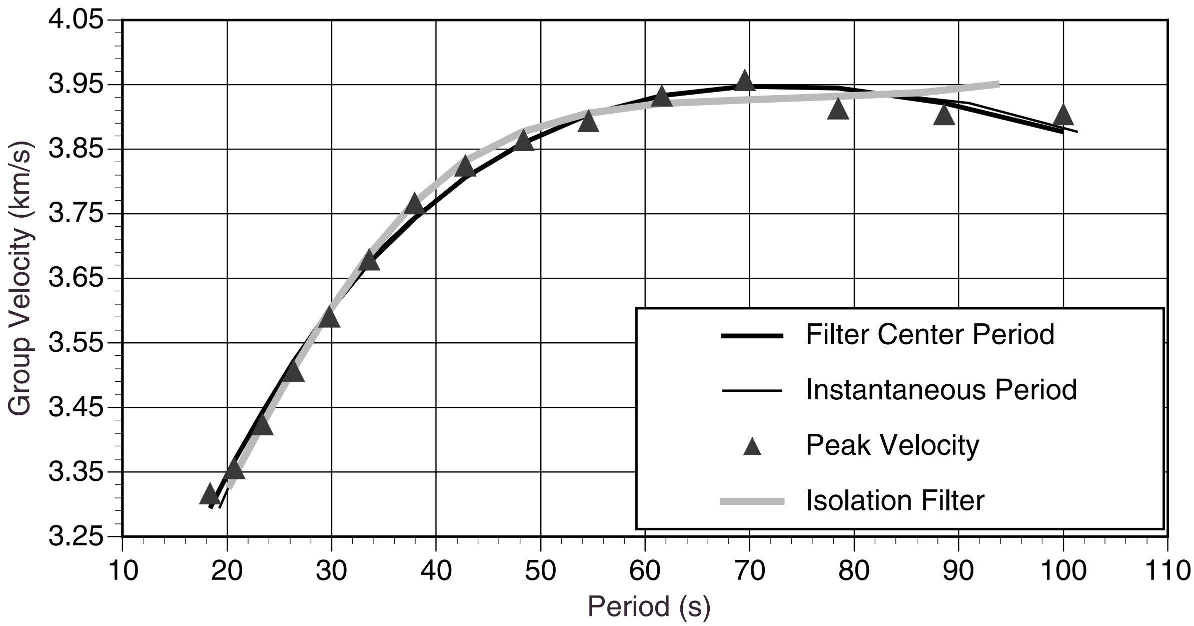


Figure 3–18. Four dispersion curves obtained from the seismogram used as an example earlier in the notes. The splined dispersion without any isolation filtering is shown as a function of filter center period and instantaneous period, the group velocities corresponding to the envelope peaks are shown as a function of filter center period, and the splined group velocity as a function of instantaneous period after isolation filtering is shown with the gray line.

the instantaneous period. The effect is visually small because the dispersion curve is flat in this period range. At times I have seen changes of tens of seconds between the actual instantaneous period and the filter center period. The effect can be critical at long periods where the spectrum may be falling off rapidly.

4 • Group Velocity Estimation Details

You can use any program more effectively if you understand the scientific and mathematical background embodied in the program's algorithms. In the next few sections I review the ideas used to estimate the group velocities. The most basic equation governing the system is the equation for "group" arrival time, t_g. This is simply the distance, traveled by the wave, Δ, divided by the group velocity $U(\omega)$, less the time the earthquake or event occurred.

$$t_g(\omega) = \frac{\Delta}{U(\omega)} - t_{origin} \tag{4–17}$$

The real observations that we make are not group velocities, but group delays. We will assume that the propagation distance is the same as the great-circle distance between the source and the receiver to compute a velocity. It's instructive to examine the above equation to test sensitivity of group velocities to errors in time and errors in distance. Let's assume that we have a source-receiver distance of 3500 *km* and we'll use a velocity of 3.5 *km/s* for some simple calculations. Suppose that we sample the signals every 2 seconds. What is the uncertainty produced by a two-second travel time error? First, the group travel time for these parameters is

$$\begin{aligned} t_g &= \frac{\Delta}{U(\omega)} = \frac{3500}{3.5} = 1000s \\ \delta U &= U_0 - \frac{\Delta}{t_g - \delta t} = 3.5 - \frac{3500}{998} = -0.07 km/s \\ \delta U &= U_0 + \frac{\Delta}{t_g - \delta t} = 3.5 - \frac{3500}{1002} = 0.07 km/s \end{aligned} \tag{4–18}$$

Another way to look at the problem is to ask what is the time error that will produce a 3% variation in the estimated group velocity? The equation that we need is

$$\delta t = t_g - \frac{\Delta}{v - \delta v} \tag{4–19}$$

For our example,

$$\delta t = 1000 - \frac{3500}{3.5 + 0.105} = 29s \qquad (4\text{–}20)$$

We can also ask what the effects of uncertainty in distance are on our measurements. For example let the true distance be Δ and the assumed distance be Δ', a 10 *km* over-estimate of source-receiver distance will give rise to an uncertainty of

$$\delta U = \frac{|\Delta - \Delta'|}{\Delta} v = \frac{|3490 - 3500|}{3490} \cdot 3.5 = 0.010 km/s \qquad (4\text{–}21)$$

for a distance of 1500 *km*, the same distance error will produce a velocity uncertainty of

$$\delta U = \frac{|\Delta - \Delta'|}{\Delta} v = \frac{|1490 - 1500|}{1490} \cdot 3.5 = 0.023 km/s \qquad (4\text{–}22)$$

Gaussian Filtering

If we band-pass filter the seismogram into narrow frequency bands, we can estimate the group arrival time by measuring the peak of the envelope of the filtered trace. The optimal filter for these analyses is a Gaussian filter, usually parameterized by a width factor α and the center frequency, ω_o. The filter is described by

$$G(\omega) = \sqrt{\frac{\alpha\pi}{\omega_0^2}} \cdot \exp\left\{-\alpha\frac{(\omega - \omega_0)^2}{\omega_0^2}\right\} \qquad (4\text{–}23)$$

For a range of periods (sampled logarithmically) I compute ω_0 and then filter the seismogram and sample the envelope (instantaneous amplitude) of the seismogram evenly in group velocity. The resulting image is contoured and the peak amplitudes for each period are identified. At this stage, the user may interactively limit the band width of the analysis or edit specific group velocity estimates.

The Gaussian Width Parameter. You have a choice of using a constant Gaussian width parameter that is entered through the file in.swmfa, or to use a distance-dependent value chosen to minimize the area of the fundamental mode surface wave on the group-velocity period spectral image. If we accept that the value of *alpha* that minimizes the area of a dispersed wavetrain on the group-velocity period image, then the choice for the parameter is

$$\alpha_{opt} = \frac{\omega}{2} \cdot \frac{dt_g}{d\omega} \tag{4–24}$$

If we define the wave slowness, $S(\omega)$, as the inverse of the group velocity, then $t_g = \Delta \times S(\omega)$ and we can rewrite Equation (4–24) as

$$\begin{aligned} \alpha_{opt} &= \frac{2\pi f\Delta}{2} \cdot \frac{dS(\omega)}{d\omega} \\ &= \frac{\pi\Delta}{T} \cdot \frac{dS(\omega)}{d\omega} \end{aligned} \tag{4–25}$$

We can convert the derivative with respect to ω into one involving the period using the chain rule:

$$\begin{aligned} \alpha_{opt} &= \frac{\pi\Delta}{T} \cdot \frac{dS(T)}{dT} \cdot \frac{dT}{d\omega} \\ &= \frac{\pi\Delta}{T} \cdot \frac{dS(T)}{dT} \cdot \frac{d(2\pi/\omega)}{d\omega} \\ &= \frac{\pi\Delta}{T} \cdot \frac{dS(T)}{dT} \cdot \frac{-2\pi}{(2\pi f)^2} \\ &= \frac{T\Delta}{2} \cdot \frac{dS(T)}{dT} \end{aligned} \tag{4–26}$$

We have to put some minimal limit on the values to avoid a zero value when the slowness does not change with period. I show these parameters for PREM Rayleigh waves in Figure 4–20. In practice, a simplified, and modified curve is used: The short-period values are replaced by the value at 25 seconds, and higher than 60s the α is constrained to slowly decrease. The shape of the curves for three different distances are shown in Figure 4–19. The variation of α results in a narrower ridge in the group-velocity versus period. For clean, simple data, the estimated velocities were not substantially different, but the *ad hoc* uncertainty measures are reduced.

Envelopes and Instantaneous Periods

One problem with the simple algorithm of band-pass filtering is that complexities in the seismogram spectrum may cause the frequency of a filtered signal to have a frequency that is not equal to the central period in the filter. That is, the dominant period in the signal after filtering is not equal

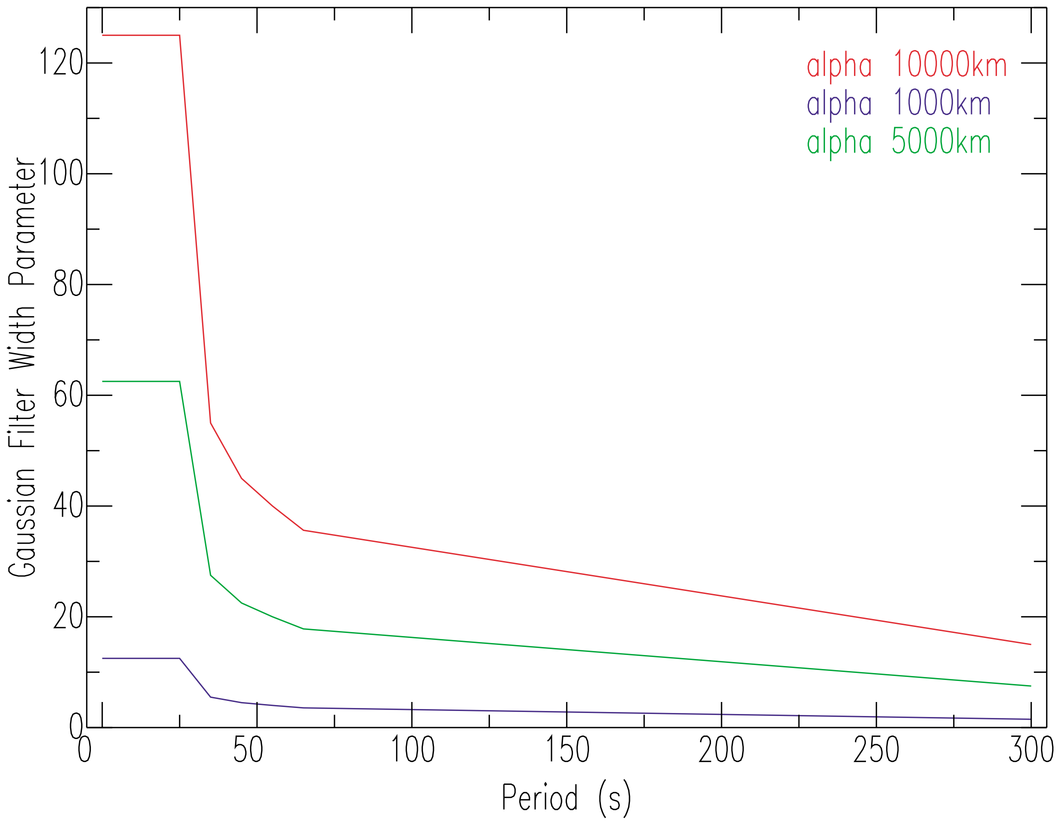


Figure 4–19. Values of α that are optimal for Rayleigh waves traveling in PREM. The values for three different distances are shown. The only difference in the α-curves is a scale factor (the values are proportional to the propagation distance.

to the filter used to construct the filter. The result can cause problems with the estimated group velocities in the vicitnity of a rapid change in the spectral amplitude. One way to minimize this problem is to directly measure the frequency or period of the signal at the same time that you measure the group velocity. Specifically, we can compute the instantaneous frequency from the signal and measure the instantaneous frequency at the peak (or splined in our case) group velocity. This method is not fool proof, and you can show with synthetic seismograms that when the signal spectrum is changing amplitude rapidly, you may not recover the correct group velocity, even when you use a measure of the instantaneous frequency. The problems are alleviated on long paths, but the errors can be substantial on short paths. The best way to get some feeling for the problem is to work with synthetic seismograms. The instantaneous period can be computed from the instantaneous frequency, which can be calculated at the same time that we compute the signal envelope.

The Analytic Signal

Claerbout(1992) has a nice, succinct description of the analytic signal, envelopes, and instantaneous frequency. I used his algorithm to compute these parameters. The analytic signal is constructed from the original seismogram and its Hilbert Transform. You can compute the Hilbert Transform by convolving the signal with an approximation of $1/\sqrt{t}$. This is the way that SAC does it's Hilbert Transform and envelope calculations and the approach is preferred if you have substantial offsets near the ends of your signals. Another approach is to use the Fourier Transform. To get the analytic signal you compute the Fourier Transform of the original signal, zero the negative frequency components, double the positive frequency components, and inverse transform (make sure you deal with the scaling of the FFT routine). The resulting time series is complex, and called the analytic signal. The magnitude of this signal as a function of time is the seismogram envelope.

$$a(t) = u(t) + iv(t) \qquad \textit{(4–27)}$$

The phase of the analytic signal is the integral of the instantaneous frequency, or

$$\phi(t) = \operatorname{atan}\left\{\frac{u(t)}{v(t)}\right\} \qquad (4\text{–}28)$$

and the instantaneous frequency is given by the temporal derivative of the phase

$$f_i(t) = \frac{d\phi}{dt}. \qquad \textit{(4–29)}$$

Computing the phase using the inverse tangent function is not a good idea because the values are wrapped by the multi-valued inverse tangent function. An alternative approach is to use the following

$$a(t) = re^{i\phi} \qquad \textit{(4–30)}$$

$$\log a(t) = \log|r| + i\phi \qquad (4\text{–}31)$$

which leads to

$$\phi(t) = \Im\{\log at\}, \qquad (4\text{–}32)$$

where $\Im\{\ \}$ indicates the imaginary part of a complex number. So,

$$\begin{aligned}
\frac{d\phi(t)}{dt} &= \frac{d}{dt}[\Im\{\log a(t)\}] \\
&= \Im\left\{\frac{d}{dt}[\log a(t)]\right\} \\
&= \Im\left\{\frac{d}{dt}[\log a(t)]\right\} \\
&= \Im\left\{\frac{1}{a(t)}\frac{da(t)}{dt}\right\} \\
&= \Im\left\{\frac{a^*(t)}{a^*(t)}\frac{\frac{da(t)}{dt}}{a(t)}\right\}
\end{aligned} \tag{4–33}$$

We can construct a stable estimate of the instantaneous frequency by performing the above computations and smoothing the complex numerator and real denominator. Following the suggestion of Claerbout (1992), I use a three-point, 1-2-1, smoothing operator and an example calculation is shown in Figure 4–20.

For most of the spectrum, discrepancies between the instantaneous period from the filter center period is small. It can be important in regions where the seismogram spectrum has a notch (such as that caused by depth in Rayleigh-wave spectra) or where the long- or short-period signals are falling off rapidly due to source excitation or attenuation effects. In these bands of rapid spectral fluctuation, the spectrum asymmetry may bias the filtered signal away from the Gaussian filter center period - using the instantaneous period removes this bias.

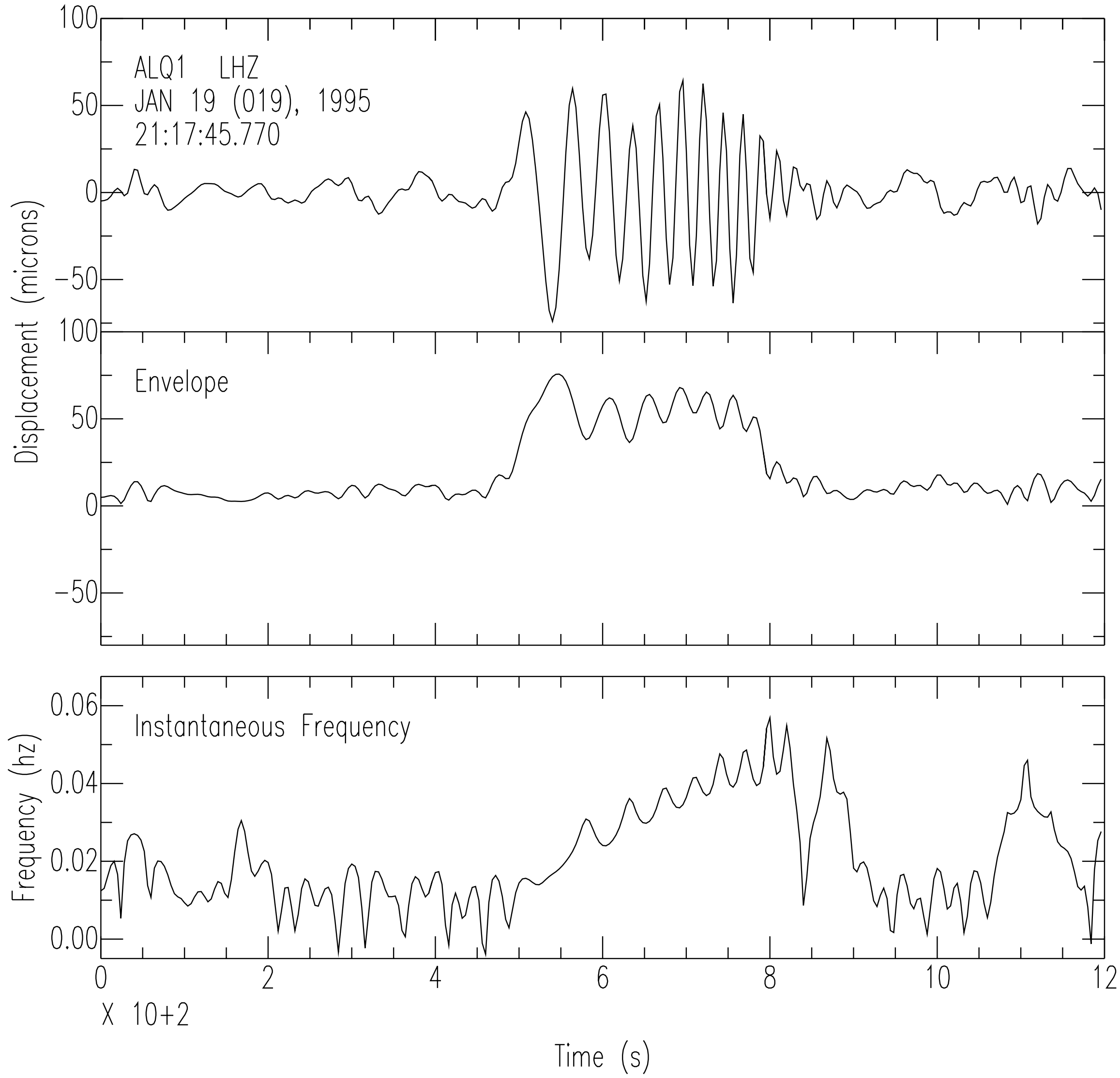


Figure 4–20. The top panel shows the original seismic displacement in microns, the middle is the envelope, and the lower panel is the instantaneous frequency. The Frequency is unstable in regions where the envelope is near zero (no signal) but stabilizes in the region of the surface waves. In the multiple filter analysis, the variation in frequency is limited by the narrow band filters.

5 • Single-Station Phase-Velocity Estimation

The relationship for the observed phase of a seismic surface wave can be expressed in terms of an initial phase from the source acted upon by a sequence of linear filters. We represent the source phase as $\phi_s(\omega)$, and the observed phase as $\phi_o(\omega)$, where $\omega = 2\,\pi f$ is the radian frequency. The two are related by

$$f_{o}(w)=f_{s}(w)+f_{p}(w)+f_{i}(w) \qquad (5–34)$$

where $\phi_i(\omega)$ is the instrument phase, and the propagation phase, $\phi_p(\omega)$, is given by

$$f_{p}(w)=\backslash\text{frac } w\ rc(w) \qquad (5–35)$$

I assume that the instrument response is known so the instrument phase in ([Ref: eq:eqphase]) can be computed and so I ignore it from this point onward. I also assume that the source phase is known well enough to provide source corrections. This is a major assumption for the resulting models and some errors are likely introduced into the data because of uncertainties in the source constraints. The multiplicity of the data will limit the influence of source uncertainty but note that careful interpretation of features in the model constrained by few data may be suspect due to inadequate source corrections. The remaining term in the expression depends on the phase velocity of the structure. I denote the source and instrument corrected phase as

$$\backslash\text{hat } \phi_{p}(\omega)=\backslash\text{frac } \omega\ rc(\omega) \qquad (5–36)$$

The inversion of ([Ref: eq:correctedpphase]) for an estimate of the phase velocity, \hat $c(\omega)$, is non-unique since we may add or subtract a factor of $2n\pi$ to the observed phase without changing the observed waveform.

$$\backslash\text{hat } c(\omega)=\backslash\text{frac } \backslash\text{hat } \phi_{p}(\omega)+2\,n\pi\ \omega\ r \qquad (5–37)$$

where n is an integer. Equations ([Ref: eq:eqphase]) through ([Ref: eq:phsvel]) dictate the basic processing sequence for the estimation of a phase velocity curve using a single station, known source mechanism. Examination of the relative size of the various phase terms can help develop an intuitive feel for the sensitivity of the phase velocity estimates to uncertainties in the source phase correction. The phase correction for the source depends on the local structure and the orientation of the source or the elements of the moment tensor. The propagation phase can be quite large, assuming phase velocities between 2.8 and 4.5 km/s and propagation distances of approximately 300 to 1500 km results in phase propagation phase corrections from ($\omega \approx 65\ s^{-1}$) to ($\omega \approx 500\ s^{-1}$). The source phase is usually much smaller but there is a frequency dependence. For short paths and longer periods, the source correction becomes a more significant fraction of the total phase and the results depend more on good source corrections.

Appendices

PGPLOT Graphics Library

The codes documented here use the graphics package developed by Dr. T. J. Pearson of Caltech. The codes are available from:

```
ftp://astro.caltech.edu/pub/pgplot/
```

and online documentation can be found at

```
http://astro.caltech.edu/ tjp/pgplot/
```

The software is protected by copyright:

REALFT Scaling

In many of these programs I make use of the subroutine *realft* to compute the Fourier Transform of the signal. This routine is part of the Numerical Recipes Package (Press *et* al., 1986). The sign convention for the transforms in Numerical Recipes is

$$H(f) = \int_{-\infty}^{\infty} h(t)e^{2\pi ift}dt \qquad (5\text{–}38)$$

$$h(t) = \int_{-\infty}^{\infty} H(f)e^{-2\pi ift}df \qquad (5\text{–}39)$$

where f is frequency and t is time. The simplest way to unravel the scaling of *realft* is to test it. The results of a test are shown below. I input a unit area time series and examine the Fourier Transform. To compute the correct area $F(\omega = 0)$ we must scale the transform by the factor *dt*. To reproduce the original time series after inverse transformation, we must then scale the output of *realft* (with an inverse flag) by $2 \cdot df$. The factor of two arises from the fact that *realft* uses a trick to compute the transform of a real time series with half the time and storage normally used in complex FFT algorithms. Here is a listing of the test program:

```
      program realft_test
      integer mxpts
      parameter(mxpts=8)
      real x(mxpts)
      integer i,halfpts,forward,inverse,stdin,stdout,stderr
       stdin = 5
       stdout = 6
       stderr = 6
       forward = 1
       inverse = -1
       halfpts = mxpts / 2
      do 1 i = 1,mxpts
        x(i) = 0
 1    continue
      dt = 0.1
      x(2) = 1 / dt
      df = 1.0 / (real(mxpts)*dt)
      write(stdout,*)' '
      write(stdout,*) 'dt: ',dt, '    df: ',df
      write(stdout,*)' '
      write(stdout,*)'Original (Unit Area)'
      write(stdout,*)' '
      do 3 i = 1,mxpts
        write(stdout,50) x(i)
 3    continue
50    format(f16.9)
```

```
      write(stdout,*)' '
      write(stdout,*)'Forward '
      write(stdout,*)' '
      write(stdout,*)' Unscaled | Scaled by * dt'
      write(stdout,*)' '
      call realft(x,halfpts,forward)
      write(stdout,100) x(1), 0.0, dt*x(1),0.0
      do 10 i = 2, halfpts-1
        j = 2 * i
        write(stdout,100) x(j), x(j+1),dt*x(j),dt*x(j+1)
10    continue
      write(stdout,100) x(2),0.0, dt*x(2),0.0
100   format(f16.9,2x,f16.9,2x,'|',2x, f16.9,2x,f16.9)
      write(stdout,*)' '
      write(stdout,*)'Inverse '
      write(stdout,*)' '
      write(stdout,*)' Unscaled | Scaled by dt * 2 * df'
      write(stdout,*)' '
      call realft(x,halfpts,inverse)
      do 20 i = 1,mxpts
        write(stdout,200) x(i), x(i) * 2*dt*df
20    continue
200   format(f16.9,2x,'|',2x,f16.9)
      stop
      end
```

and here is the output:

```
dt:     1.00000E-01    df:     1.25000
Original (Unit Area)
    0.000000000
   10.000000000
    0.000000000
    0.000000000
    0.000000000
    0.000000000
    0.000000000
    0.000000000
Forward
 Unscaled | Scaled by * dt
   10.000000000       0.000000000  |       1.000000000       0.000000000
    7.071067810       0.000000000  |       0.707106769       0.000000000
   10.000000000      -7.071067810  |       1.000000000      -0.707106769
  -10.000000000       0.000000000  |      -1.000000000       0.000000000
Inverse
 Unscaled | Scaled by dt * 2 * df
    0.000000000  |       0.000000000
   40.000000000  |      10.000000000
    0.000000000  |       0.000000000
    0.000000000  |       0.000000000
    0.000000000  |       0.000000000
    0.000000000  |       0.000000000
    0.000000000  |       0.000000000
    0.000000000  |       0.000000000
```

Dispersion Curves

This appendix consists of tables of dispersion curves commonly cited in the literature. The point is to collect this information in one location for convenience.

PREM - Anderson and Dziewonski (PEPI, 1981)

To construct this table, I took the original dispersion file from Yushen Zhang into SAC and interpolated at five-seconds period.

	Rayleigh		Love	
Period	**Group Velocity**	**Phase Velocity**	**Group Velocity**	**Phase Velocity**
35	3.784	3.913	4.077	4.400
45	3.832	3.940	4.243	4.469
55	3.838	3.962	4.312	4.512
65	3.827	3.985	4.344	4.545
75	3.807	4.010	4.362	4.574
85	3.784	4.039	4.372	4.602
95	3.763	4.071	4.378	4.629
105	3.743	4.106	4.382	4.655
115	3.724	4.145	4.384	4.682
125	3.706	4.186	4.385	4.709
135	3.690	4.229	4.385	4.736
145	3.673	4.276	4.385	4.763
155	3.657	4.325	4.384	4.790
165	3.641	4.376	4.384	4.818
175	3.625	4.430	4.383	4.847
185	3.610	4.486	4.382	4.875
195	3.596	4.546	4.382	4.905
205	3.583	4.608	4.381	4.934
215	3.574	4.673	4.381	4.964
225	3.569	4.741	4.382	4.994
235	3.568	4.810	4.383	5.025
245	3.573	4.882	4.385	5.055
255	3.584	4.955	4.387	5.087
265	3.603	5.030	4.390	5.118
275	3.630	5.104	4.394	5.149
285	3.663	5.179	4.399	5.181
295	3.704	5.254	4.404	5.213
305	3.752	5.327	4.410	5.245
325	3.865	5.470	4.426	5.309
345	3.995	5.605	4.445	5.374
365	4.132	5.731	4.468	5.438
385	4.268	5.847	4.495	5.501
405	4.400	5.955	4.525	5.564
425	4.526	6.054	4.558	5.626
445	4.647	6.146	4.594	5.686
465	4.764	6.232	4.633	5.746
485	4.885	6.311	4.675	5.804
505	5.015	6.383	4.718	5.860

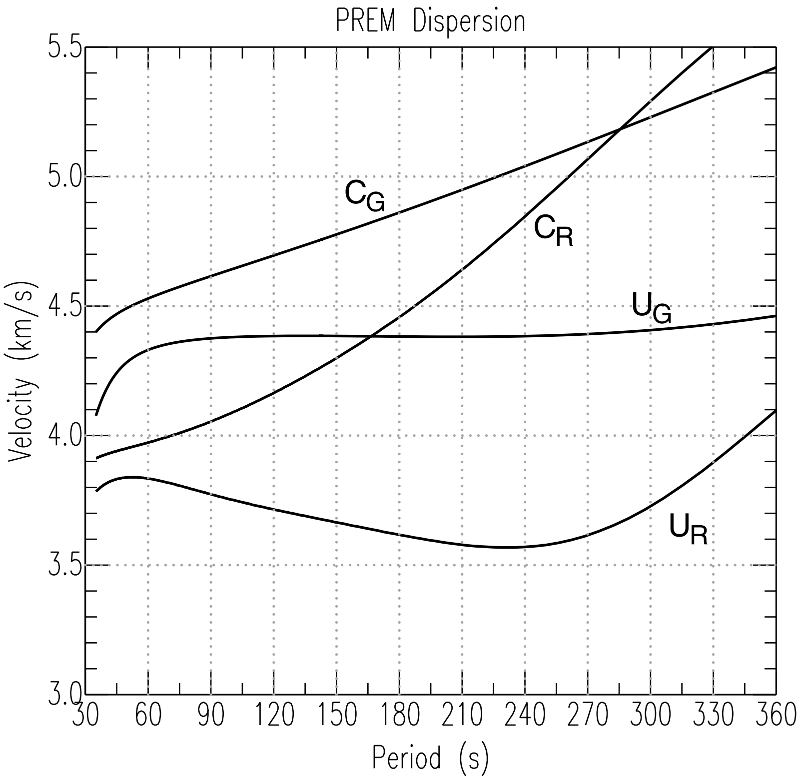


Figure 5–21. Dispersion values for the PREM earth model.

Canadian Shield Dispersion - Brune and Dorman (1963)

Period	Phase Velocity	Phase Velocity
10	3.35	3.75
12	3.40	3.79
14	3.46	3.84
16	3.52	3.90
18	3.60	3.95
20	3.67	4.01
22	3.74	4.06
24	3.81	4.11
26	3.87	4.16
28	3.92	4.21
30	3.96	4.25
32	3.99	4.29
34	4.01	4.32
36	4.03	4.35
38	4.05	4.38
40	4.06	4.40
42	4.07	4.42
44	4.08	4.43
46	4.09	4.45
48	4.09	4.47
50	4.10	4.48
52	4.10	4.49
54	4.11	4.50
56	4.11	4.51
58	4.11	4.51
60	4.12	4.52
62	4.12	
72	4.13	
80	4.14	
88	4.15	
100	4.16	
110	4.18	
120	4.20	
130	4.22	

Moment-Tensor Conventions

To convert from Howard Patton's convention to Aki and Richards Convention you have to switch two diagonal elements (11 and 22) and then switch elements 13 and 12 and change their sign. The result is an Aki and Richards moment tensor.

$$\begin{bmatrix} m_{ee} & m_{en} & m_{eu} \\ & m_{nn} & m_{nu} \\ & & m_{uu} \end{bmatrix} = \begin{bmatrix} m_{nn} & m_{ne} & -m_{nu} \\ & m_{ee} & -m_{eu} \\ & & m_{uu} \end{bmatrix} \tag{5–40}$$

The relationship between Patton's convention and Harvard's CMT coordinates is summarized by

Harvard	**Patton**
M_{RR}	M_{UU}
$M_{R\Delta}$	$-M_{NU}$
$M_{R\Phi}$	M_{EU}
$M_{\Delta\Delta}$	M_{NN}
$M_{\Delta\Phi}$	$-M_{NE}$
$M_{\Phi\Phi}$	M_{EE}