

MATLAB Exercise • Level 1

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Elastic Constants from Vp, Vs, & Density

COMPUTING ELASTIC CONSTANTS

We often discuss elastic materials of rocks by specifying the compressional velocity, V_p , the shear-wave velocity, V_s , and the density. Sometimes however, it is useful to know the corresponding values of Lamé's parameters (λ , μ), or the bulk or Young's modulus, or Poisson's ratio. Another useful quantity is the V_p/V_s ratio, which of course is trivial to compute. In this exercise we will write a MATLAB script that accepts the seismic wave velocities and density as inputs and returns the values of the elastic constants.

The formulas that we need are

$$\begin{aligned} V_p &= \sqrt{\frac{\lambda + 2\mu}{\rho}} \\ V_s &= \sqrt{\frac{\mu}{\rho}} \end{aligned} \quad (1)$$

where λ and μ are Lamé's parameters, and ρ is the density. We can invert these equations to find Lamé's parameters

$$\begin{aligned} \mu &= \rho \cdot V_s^2 \\ \lambda &= (V_p)^2 \cdot \rho - 2\mu \end{aligned} \quad (2)$$

If you convert the velocities to MKS units before computing the values, then the units of Lamé's parameters are Pascals.

Once we have the values of Lamé's parameters, we can use the following formulas to compute the other moduli and ratios

$$\begin{aligned} K &= \lambda + \frac{2\mu}{3} \\ E &= \frac{9K\lambda}{3K + \mu} \\ \sigma &= \frac{\lambda}{2 \cdot (\lambda + \mu)} \end{aligned} \quad (3)$$

where K is the bulk modulus, E is Young's modulus, and σ is Poisson's ratio.

The hardest part of constructing a valuable script to compute these numbers will be keeping track of the units. For Young's modulus and Poisson's ratio, this won't matter (why?). Here's the script that I wrote to do the calculation. I've deleted one of the computations (Poisson's ratio) because I want you to type in the script and supply the omitted code.

```
function [ lamda, mu, BM, YM, PR, VPVS ] = econstants(vp,vs,rho)
%
% Function to compute elastic constants from seismic velocities
% and density.
%
% INPUT:
%   vp = P-velocity in km/s
%   vs = S-Velocity in km/s
%   rho = density in g/cm^3
%
% OUPUT:
%   lamda, mu (Lame's parameters in Pascals)
%   BM (Bulk modulus in Pascals)
%   YM (Young's modulus)
%   PR (Poisson's ratio)
%   VPVS (the ratio of Vp over Vs)
%
%
% convert velocity & density to MKS units
vp = vp * 1000;
vs = vs * 1000; % this does not affect vs in the calling function
rho = rho * 1000;
%
mu = rho*vs*vs;
lamda = vp*vp*rho - 2 * mu;
BM = lamda + (2*mu)/3;
YM = (9*BM*mu) / (3*BM+mu);
VPVS = vp/vs;
```

To use the script:

```
>> [ lamda, mu, BM, YM, PR, VPVS ] = econstants(6,6/sqrt(3),2.67)
>> lamda =
>> 3.203999999999999e+10
>> mu =
>> 3.204000000000000e+10
>> BM =
>> 5.340000000000000e+10
>> YM =
>> 8.010000000000002e+10
>> PR =
>> 0.250000000000000
>> VPVS =
>> 1.73205080756888
```

Note that the values are what is expected for a Poisson solid ($V_p/V_s = \sqrt{3}$).

EXERCISES

Exercise 1: Use dimensional analysis to show that the units of Lamé's parameters, λ and μ are Pascals (the same units as a stress or pressure).

Exercise 2: What are the units of each of the above moduli and ratios?

Exercise 3: Type in the econstants script and add the appropriate formula to compute Poisson's ratio above the line

$$VPVS = v_p/v_s;$$

Then run your script to compute the values of the constants for the following materials (the values are from Kern and Richter, J. Geophysics, Volume 49, pages 47-56, 1981)

Table 1: Seismic Velocities of Selected Rocks

Material (@ 600 MPa, 20°C)	V _p (km/s)	V _s (km/s)	Density (gm/cm ³)
Granite	5.99	3.40	2.70
Plg-amph-gneiss	6.50	3.77	2.86
Dunite	8.23	4.46	3.28
Peridotite	8.01	4.56	3.29

make a table of the results.

Exercise 4: If you assume that the density of upper crustal material is 2,700 kg/m³ and that the pressure at a depth z is given by $P = \rho g z$, where $g = 9.8(m/s^2)$, what is the depth at which the velocities in Table 1 are appropriate. Is 20°C an appropriate temperature for that depth?