

MATLAB Exercise • Level 2

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Plotting the Mohr Circle for A Stress Tensor

THE MOHR CIRCLE

This is primarily a graphics exercise that uses the results of another MATLAB Exercise: *Computing the Eigenvalues and Eigenvectors of a Symmetric Tensor*. If you haven't worked through that example, you may not follow the details of this exercise. In this exercise we take the results of a stress-tensor decomposition and plot the 2D Mohr circle corresponding to the stress state.

The equations of the circle are most easily defined in terms of the angle between the fault normal and the principal axis of stress, θ . If we plot the normal stress value on the plane along the horizontal axis and the shear stress on the plane along the vertical axis, our equations are

$$\begin{aligned}\sigma_n &= \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2} \cdot \cos 2\theta \\ \sigma_s &= \frac{\sigma_1 - \sigma_3}{2} \cdot \sin 2\theta\end{aligned}\tag{1}$$

where σ_1 and σ_3 are the maximum and minimum principal stresses. We can compute the circle by allowing the angle θ to range from 0° to 180° and calculating the values of the normal and shear stress.

ILLUSTRATING SHEAR STRESS ON A PLANE

One reason for the Mohr circle is that it allows us to visualize the state of stress on a plane and graphically compute the normal and shear stress acting on a plane. We can do the stress computation algebraically as well, but the state of stress is often clear when you use the Mohr circle. To show the value of stress on a particular plane, specified by the angle the plane's normal vector makes with the maximum principal stress direction, we draw a line from the center of the Mohr circle with an angle with the horizontal axis equal to twice θ .

To draw a line, you need is two points. We know that one point is at the center of Mohr's circle. So we compute another point with a length equal to 1.1 times the maximum shear stress (and extra 10% length insures that the line crosses the circle). The values for the endpoints of the line are

$$\begin{aligned} &(\bar{\sigma}, 0) \\ &(\bar{\sigma} \cdot \cos(2\theta), \bar{\sigma} \cdot \sin(2\theta))\end{aligned}\tag{2}$$

where

$$\bar{\sigma} = \frac{\sigma_1 + \sigma_3}{2}. \quad (3)$$

We can draw the line and read off the value of the normal and shear stresses. Of course we could compute them as well (I'll leave that as an exercise).

COULOMB FAILURE ENVELOPE

We can add one last item to our illustration - a Coulomb failure envelope. Again, you should look at a structural geology text to review the theory, but the Coulomb failure criteria is a linear equation with a slope given by the coefficient of static friction, f_s . In other words, a fault will fail if the surface experiences at shear stress of

$$|\sigma_s| = f_s \cdot \sigma_n. \quad (4)$$

The value of f_s is typically about 0.85 (which is what I assume in the script below), but it can vary somewhat.

THE SCRIPT MOHR2D.M

Here's my script to perform the above computations. Note that I use the function *stensor_decomp* to compute the principal values of the input stress tensor.

```
function [V, D] = mohr2D(S,angle)
%
% function to decompose a symmetric tensor, S
%
% THIS FUNCTION REQUIRES THE FUNCTION stensor_decomp(S)
%
% INPUT:
%   the tensor, S in the form:
%
%       | Snn Sne Snd |
%       | Sen See Sed |
%       | Sdn Sde Sdd |
%
%   where n = north, e = east, and d = down
%
% into principal values and principal axes
%
% OUTPUT:
%   matrices V (columns are eigenvectors)
%           D (diagonal with principal values)
%
% Example:
%
%   >> S = [100,-50,0;-50,150,-35;0,-35,30];
```

```

%     >> [V D] = stensor_decomp(S);
%

% Call stensor_decomp to decompose the stress tensor
%
[V, D] = stensor_decomp(S);
%
%
% Draw the Mohr Circle
%
mean_normal = 0.5 * (D(1,1) + D(3,3));
max_shear   = 0.5 * (D(1,1) - D(3,3));
%
theta = (0:180) * pi / 180;
ns = mean_normal + max_shear * cos(2*theta);
ss = max_shear * sin(2 * theta);
%
hm = gca;
% plot the circle
hl = plot(ns,ss,'k-','Color',[.25 .25 .25],'LineWidth', 1.5);
%
% plot the center of the circle and the origin
%
hold on;
plot(mean_normal,0,'k.','MarkerSize',18,'Color',[.25 .25 .25])
plot(0,0,'k.','MarkerSize',24,'Color',[.25 .25 .25])
grid on;
%
% plot the line (if an angle is given)
%
% go from the origin to one other point
%   that with a normal stress equal to the maximum
%
%
if nargin > 1
hold on;
xl(1) = mean_normal;
yl(1) = 0;
mylength = 1.1 * max_shear;
xl(2) = xl(1) + mylength * cos(2*angle*pi/180);
yl(2) = yl(1) + mylength * sin(2*angle*pi/180);
%
plot(xl, yl,'k-','LineWidth',1.25);
plot(xl,-yl,'k-','LineWidth',1.25);
mytitle = sprintf('Angle = %.1f Degrees', angle)
title(mystring);
end
%
% Plot a Coulomb Envelope (mu = 0.85)
%
hold on;
mu = 0.85;
xl(1) = 0;
yl(1) = 0;
mylength = 1.5 * max_shear;
xl(2) = xl(1) + mylength;
yl(2) = yl(1) + mylength * mu;
plot(xl, yl,'k-','LineWidth',1.25);
plot(xl,-yl,'k-','LineWidth',1.25);

```

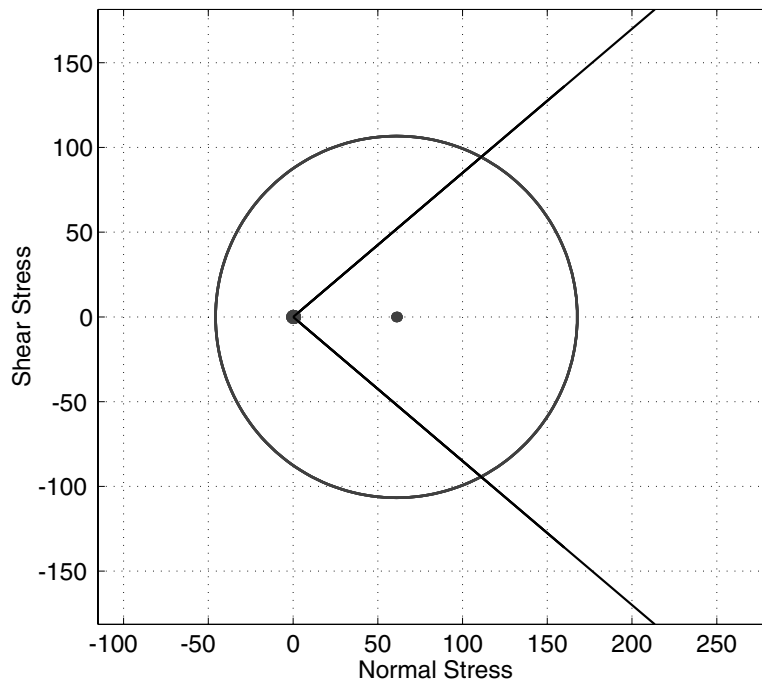


Figure 1. Mohr circle and Coulomb envelope. The large point identifies the coordinate origin and the smaller dot identifies the center of the circle. The physical units of stress depend on the input values in the stress tensor.

```
%
% add some labels
%
set(hm,'FontName','Helvetica','FontSize',14);
xlabel('Normal Stress');
ylabel('Shear Stress');
axis square;axis equal;
hold off;
```

Figure 1 contains the results of an example computation, with the stress tensor given by

$$\begin{bmatrix} -10.3 & 0 & 79.4 \\ 0 & 39.7 & 0 \\ 79.4 & 0 & 132.3 \end{bmatrix} MPa \quad (5)$$

In MATLAB, I executed the following commands

```
>> S = [-10.3,0,79.4;0,39.7,0;79.4,0,132.3]
>> S =
>> -10.3000    0    79.4000
>>    0    39.7000    0
>>    79.4000    0    132.3000
>>
```

```

>> mohr2d(S);
>>
>> The eigenvalues are:
>>
>>     167.71   39.70  -45.71
>>
>> The eigenvectors are:
>>
>>         V1      V2      V3
>>
>> N:  0.407   0.000   0.913
>>
>> E:  0.000   1.000   0.000
>>
>> D:  0.913   0.000  -0.407

```

The text output is a result of the *stensor_decomp* routine.

EXERCISES

Exercise 1: Plot the Mohr Circle for

$$S = \begin{bmatrix} 90 & -20 & 2 \\ -20 & 45 & -5 \\ 2 & -5 & 3 \end{bmatrix}$$

At what angle with a slip occur on a fault given the stress tensor above?

Exercise 2: Plot the Mohr Circle for

$$S = \begin{bmatrix} 2 & -20 & 2 \\ -20 & 45 & -5 \\ 2 & -5 & 100 \end{bmatrix}$$

At what angle with a slip occur on a fault given the stress tensor above?

Exercise 3: Plot the Mohr Circle for

$$S = \begin{bmatrix} 2 & -20 & 2 \\ -20 & 45 & -5 \\ 2 & -5 & -100 \end{bmatrix}$$

At what angle with a slip occur on a fault given the stress tensor above?