

# MATLAB Exercise • Level 2

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## Computing Local Plate Velocities

### COMPUTING RELATIVE PLATE MOTIONS

The theory behind the following calculations is described nicely in Chapter 4 of Cox and Hart, *Plate Tectonics, How It Works*, Blackwell Publishers, 1986. The basic idea of this exercise is to use information on plate motions that are contained in “plate-motion models” to compute the “local” velocity between two plates. By “local” I mean that you specify a location on one plate and compute the velocity of that point with respect to another, specified plate. For example, we may want to know the convergence direction and rate between two plates,  $A$  and  $B$ . Specifically, we might want to know that velocity of a point on plate  $B$  has a velocity with respect to plate  $A$  given by (velocity is a vector)

$${}_A V_B = 0.5 \text{ cm/yr North} + 2.0 \text{ cm/yr East} . \quad (1)$$

In most plate motion models, the information on the relative plate motion between a pair of plates is summarized in a rotation axis (vector) that intersects Earth’s surface at a point called the Euler Pole of Rotation (for those two plates). Each pair of plates has a distinct rotation axis and a plate-motion model is generally summarized by specifying the rotation axes of all the plate pairs (or enough to allow the calculation of all the pairs). We generally specify the axis by listing its intersection with Earth’s surface using latitude and longitude along with a rate value,  $\omega$ , which contains information on the relative speed of the plates.

To compute the velocity at a point,  $p$ , on Earth’s surface and on plate  $B$ , with respect to plate  $A$ , we need the appropriate information on the rotation axis, and the latitude and longitude of the point of interest ( $p$ ). Then we can calculate

$$v = {}_A \vec{\omega}_B \times (R \vec{p}) \quad (2)$$

where  ${}_A \vec{\omega}_B$  is the plate rotation vector,  $\vec{p}$  is the position vector of our point of interest, and  $R$  is the radius of Earth. To compute the needed vector cross product, it is easier to convert the latitudes and longitudes to vectors into a Cartesian Reference frame centered at Earth’s center. If we refer to points on Earth’s surface by the latitude,  $\lambda$  and longitude,  $\phi$ , we convert to a Cartesian system using:

$$\begin{aligned}
p_x &= \cos\lambda \cdot \cos\phi \\
p_y &= \cos\lambda \cdot \sin\phi \\
p_z &= \sin\lambda
\end{aligned} \tag{3}$$

The  $x$ -component lies in the plane of Earth's equator and points toward a longitude of  $0^\circ$ , the  $y$ -component is in the equatorial plane and points towards  $90^\circ$  East, and the  $z$ -component points towards the North Pole.

We can compute the vector cross product

$$\vec{v} = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = R \cdot \begin{pmatrix} \omega_y \cdot p_z - p_y \cdot \omega_z \\ \omega_z \cdot p_x - p_z \cdot \omega_x \\ \omega_x \cdot p_y - p_x \cdot \omega_y \end{pmatrix} \tag{4}$$

which provides the velocity values *in the Cartesian coordinate system*. An easier to work with system is the local geographic system of north, east, and down. To convert  $\vec{v}$  in Cartesian coordinates to a North, East, and Down, system centered at point P, we can use

$$\begin{pmatrix} v_n \\ v_e \\ v_d \end{pmatrix} = T \cdot \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} \tag{5}$$

where

$$T = \begin{bmatrix} (-\sin\lambda)\cos\phi & (-\sin\lambda)\sin\phi & \cos\lambda \\ -\sin\phi & \cos\phi & 0 \\ (-\cos\lambda)\cos\phi & (-\cos\lambda)\sin\phi & -\sin\lambda \end{bmatrix}. \tag{6}$$

Once we have the velocity in local coordinates we can compare convert it to a magnitude and direction specified by the azimuth. Once we have the velocity in local coordinates we can compute the magnitude of motion

$$v = \sqrt{v_n^2 + v_e^2 + v_d^2} \tag{7}$$

and the azimuth of the motion

$$\theta = \text{atan} \left\{ \frac{v_e}{v_n} \right\}. \tag{8}$$

Be careful with the tangent function since it is multi valued. You have to check to be sure it's in the appropriate quadrant, and be sure to measure it clockwise from north.

## MATLAB SCRIPT PLATE\_VEL

Here's the script that I wrote to perform the calculations through equation (6). Enter this script into a file called *plate\_vel.m* and then place the file into a directory in your MATLAB path.

```
function vl = plate_vel(lat, lon, latp, lonp, omega)
%
% function to compute local plate velocity vector
% given the position and the Euler pole position and
% angular velocity.
%
% INPUT:
%
% lat, lon - position of the point where you want velocity
% latp, lonp, omega - Euler pole position and velocity
%
% latitudes and longitudes are in degrees
% omega is in degrees per million years
%
% OUTPUT:
%
% vl = [vn, ve, vd]' velocity in north, east, and down directions
% (referred to point p) in mm/yr.
%

Re = 6370.8e6; % Radius of Earth in millimeters
%
% convert from degrees to radians (original values are unchanged)
%
deg_to_rad = pi / 180;
lat = lat *deg_to_rad;
lon = lon *deg_to_rad;
latp = latp*deg_to_rad;
lonp = lonp*deg_to_rad;
%
omega = omega * 1e-06 * (pi/180); % Convert to radians per year
%
% convert to Cartesian Coordinates
%
P = [ cos(lat)*cos(lon), cos(lat)*sin(lon), sin(lat) ]';
EP = [cos(latp)*cos(lonp), cos(latp)*sin(lonp),sin(latp)]' * omega;
%
VC = Re * cross(EP,P); % compute the cross product: EP x P
%
% Rotate to local coordinate system (Cox & Hart Box 4-2);
%
T = zeros(3,3);
T(1,1) = -sin(lat)*cos(lon);
T(1,2) = -sin(lat)*sin(lon);
T(1,3) = cos(lat);
%
T(2,1) = -sin(lon);
```

```

T(2,2) = cos(lon);
T(3,2) = 0;
%
T(3,1) = -cos(lat)*cos(lon);
T(3,2) = -cos(lat)*sin(lon);
T(3,3) = -sin(lat);
%
v1 = T * VC;

```

Of course we'll need a plate motion model. The pertinent information contained in Fowler's, *The Solid Earth, An Introduction to Global Geophysics* (1990), is reproduced in Table 1. You must be

**Table 1: Plate Rotation Vectors (After Fowler, 1990)**

Plate Pair	Latitude (°)	Longitude (°)	Angular Velocity (°/Ma)
Africa-Antarctica	5.6N	-39.2E	0.13
Africa-Eurasia	21.0N	-20.6E	0.13
Africa-North America	78.8N	38.3E	0.25
Africa-South America	62.5N	-39.4E	0.32
Australia-Antarctica	13.2N	38.2E	0.68
Pacific-Antarctic	-64.3N	96.0E	0.91
South America-Antarctic	-86.4N	139.3E	0.27
Arabia-Eurasia	24.6N	13.7E	0.52
India-Eurasia	24.4N	17.7E	0.53
Eurasia-North America	62.4N	135.8E	0.22
Eurasia-Pacific	61.1N	-85.8E	0.90
Pacific-Australia	-60.1N	-178.3E	1.12
North America-Pacific	48.7N	-78.2E	0.78
Cocos-North America	27.9N	-120.7E	1.42
Nazca-Pacific	55.6N	90.1E	1.42
Nazca-South America	56.0N	-94.0E	0.76

careful how you use the information. When you read the plate pair, the value in the table is the angular velocity for computing the motion of a point on the second plate relative to a point on the first plate.

For example, the value for calculating the motion of a point on the Pacific plate relative to a point on North America is (48.7N, 78.2W,0.78). For example:

```

>> v1 = plate_vel(37,-123,48.7,-78.2,0.78)
>> v1 =
>> -40.33431624537931
>> 27.59254158624030
>> 0.000000000000000

```

The motion of the point near 37N,123W is 40 mm/yr south, and 27 mm/yr east. For a point in the Mojave Desert on North America, the motion relative to the Pacific plate is

```

>> »v1 = plate_vel(34.75,-116.5,48.7,-78.2,-0.78)
>> v1 =
>> 35.47707891158412
>> -27.93047805570016
>> -0.000000000000000

```

which indicates that the Mojave is moving to the northwest relative to the Pacific Plate.

## The Magnitude and Azimuth of Relative Motion

The magnitude of the relative plate motion is

```

>> »norm(v1)
>> ans =
>> 45.15235024357774

```

or about 45 mm/yr. The azimuth can be computed using the *atan2* function of MATLAB. Generally MATLAB references angles to the horizontal axis, in Geosciences, we use the vertical axis to represent north. So we must subtract the MATLAB *atan2* result value from  $\pi/2$  to get the result.

```

>> (pi/2 - atan2(v1(1),v1(2)))*180/pi
>> ans =
>> -38.21273418788474

```

## EXERCISES

**Exercise 1:** Write a script to take the a vector in local coordinates and return the values of the magnitude of the motion and the azimuth.

**Exercise 2:** Compute the plate motion between India and Eurasia along the Himalayan Front.

**Exercise 3:** Compute the plate motion between the Nazca and South American plates. Is the motion perpendicular to the trench?

**Exercise 4:** What is the rate that Iceland is pulling apart?

**Exercise 5:** Compare the spreading rates along the Mid-Atlantic, East-Pacific Rise, and the Red Sea. To get the motion of the Red Sea with respect to Africa, you will have to compute the angular velocity vector using the equation

$${}^A\vec{\omega}_B = {}^B\vec{\omega}_C + {}^B\vec{\omega}_A$$

and combine the information from Arabia-Eurasia and Africa-Eurasia.

**Exercise 6:** Modify the script to produce a readable text summary of the computation including the local global and local vector components, and the magnitude and azimuth of motion.

**Exercise 7:** Another way of computing the cross-product of two vectors is to use the angle between the two

$$v = {}_A\vec{\omega}_B \times R\vec{p} = |{}_A\vec{\omega}_B| \cdot |R\vec{p}| \cdot \sin\Delta$$

where  $\Delta$  is the angular distance ( $1^\circ = 111.19$  km) between the two vectors. Use this equation to determine where the velocity between two plates is maximum (how far from the Euler pole?).